Takaaki Fujita
Florentin Smarandache

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Florentin Smarandache

A Review and Introduction to Neutrosophic Applications across Various Scientific Fields

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A Review and Introduction to Neutrosophic Applications across Various Scientific Fields

Takaaki Fujita [1] * *and Florentin Smarandache* [2]

[1] Independent Researcher, Tokyo, Japan.
Email: Takaaki.fujita060@gmail.com

Abstract

A Neutrosophic Set assigns to each element independent degrees of truth, indeterminacy, and falsity, thereby extending classical uncertainty models. It is widely recognized as a generalization of both Fuzzy Sets and Intuitionistic Fuzzy Sets. Neutrosophic and fuzzy logics have been employed across sociology, psychology, the social sciences, biology, chemistry, physics, and philosophy. This paper revisits and systematizes these applications of Neutrosophic Logic, presenting rigorous mathematical definitions alongside concrete, domain–specific examples. Our aim is two fold: to promote broader understanding and dissemination of neutrosophic methods, and to provide a clear foundation that enables future theoretical development and practical deployment. By organizing existing concepts into a coherent framework and illustrating their use through reproducible constructions, we seek to broaden the scope of applications and to stimulate further research across disciplines.

Keywords: Neutrosociology, Psychology, Social Science, Sociology, Psychology, Biology, Chemistry, Physics, and Philosophy, Neutrosophic Set, Plithogenic Set

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Division of Mathematics and Sciences
University of New Mexico
705 Gurley Ave., Gallup Campus
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University of Guayaquil
Av. Kennedy and Av. Delta
"Dr. Salvador Allende" University Campus
Guayaquil 090514, Ecuador

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Maikel Leyva-Vázquez
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maikel.leyvav@ug.edu.ec

Maissam Jdid
Faculty of Science, Damascus University, SYRIA
maissam.jdid66@damascusuniversity.edu.sy

Victor Christianto
Malang Institute of Agriculture (IPM), Malang, INDONESIA
victorchristianto@gmail.com

Chapter 1: Preliminaries

This section gathers the basic notions and notation used throughout the paper. Unless explicitly stated otherwise, we work in the finite setting. By convention, the empty set is treated as an element of every set.

1.1 Neutrosophic Set with Some Examples

A Neutrosophic Set assigns to each element independent degrees of truth, indeterminacy, and falsity [1–3]. Neutrosophic sets are known to generalize fuzzy sets [4–6], intuitionistic fuzzy sets [7,8], vague sets [9,10], and picture fuzzy sets [11,12], among others. Furthermore, several extensions and related concepts of the Neutrosophic Set are known, including the Bipolar Neutrosophic Set [13–15], Quadripartitioned Neutrosophic Set [16,17], Pentapartitioned Neutrosophic Set [18–20], Heptapartitioned Neutrosophic Set [21–23], Double-valued Neutrosophic Sets [24–28], Interval-valued Neutrosophic Sets [29–32], Hesitant Neutrosophic Sets [33–35], Pythagorean Neutrosophic Sets [36–38], and q-Rung Orthopair Neutrosophic Set [39, 40].

Definition 1.1 (Neutrosophic Set)**.** [1,41] Let X be a non-empty set. A *Neutrosophic Set (NS) A* on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Example 1.2 (Email spam filtering as a Neutrosophic Set)**.** Let $X = \{\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3\}$ be three incoming emails. Define the neutrosophic set A = "is spam". For each message m_i we specify the triplet $\big(T_A(\mathrm{m}_i),\, I_A(\mathrm{m}_i),\, F_A(\mathrm{m}_i)\big) \in [0,1]^3$:

email	T_A	I_A	F_A
m_1 (bulk promo, DKIM fail)	0.85	0.10	0.12
m_2 (mixed cues, new sender)	0.50	0.35	0.40
m_3 (ongoing thread, clean headers)	0.10	0.08	0.88

Interpretation: m_1 has high evidence of being spam (large T_A) with mild uncertainty (I_A); m_2 carries conflicting features, hence higher indeterminacy; m_3 has strong evidence against spam (large F_A). All rows satisfy $0 \le T_A + I_A + F_A \le 3$.

Example 1.3 (On-road obstacle detection as a Neutrosophic Set)**.** Consider frames $X = \{\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3\}$ from a vehicle camera + LiDAR stack. Let A = "an obstacle is present in the ego lane". Assign neutrosophic triplets using fused perception scores (truth), weather/occlusion ambiguity (indeterminacy), and free-space detection (falsity):

frame	T_A	I_A	F_A
f_1 (clear day, LiDAR return ahead)	0.78	0.07	0.20
f_2 (heavy fog, partial occlusion)	0.46	0.38	0.35
f_3 (lane empty, robust freespace map)	0.12	0.10	0.90

Here, f_1 indicates a likely obstacle (high T_A) with low ambiguity; f_2 is uncertain due to adverse conditions (elevated I_A); f_3 strongly supports "no obstacle" (high F_A). Again, $0 \le T_A + I_A + F_A \le 3$ is respected for each frame.

Example 1.4 (Medical diagnosis with incomplete clinical data)**.** Let X be the set of possible conditions for a patient:

$$X = \{\text{bacterial infection},\ \text{viral infection},\ \text{allergy}\}.$$

A physician receives: (i) fever 38.5°C, (ii) elevated CRP but not very high, (iii) no travel history, (iv) patient took an unknown over–the–counter drug. Because the drug may have suppressed some symptoms, the physician cannot give fully crisp membership to any diagnosis.

Define a neutrosophic set A = "current plausible diagnosis" on X as follows:

$$\begin{aligned}
T_A(\text{bacterial infection}) &= 0.6 && \text{(fever + moderately high CRP support this)},\\
I_A(\text{bacterial infection}) &= 0.25 && \text{(drug effect and missing cultures)},\\
F_A(\text{bacterial infection}) &= 0.1 && \text{(no strong focus found)};\\
T_A(\text{viral infection}) &= 0.5 && \text{(fever could be viral)},\\
I_A(\text{viral infection}) &= 0.3 && \text{(overlap of symptoms with allergy)},\\
F_A(\text{viral infection}) &= 0.2; &&\\
T_A(\text{allergy}) &= 0.25 && \text{(mild rash)},\\
I_A(\text{allergy}) &= 0.4 && \text{(drug might have caused rash)},\\
F_A(\text{allergy}) &= 0.3. &&
\end{aligned}$$

Here, each element of X is evaluated by truth (what current data support), indeterminacy (what is unknown or confounded), and falsity (what contradicts the condition). This allows the doctor to proceed with tests while still encoding uncertainty.

Example 1.5 (Customer interest prediction in e–commerce)**.** Consider an online shop with item set

$$X = \{\text{running shoes},\ \text{wireless earbuds},\ \text{laptop backpack}\}.$$

A user browses several pages, clicks on "running shoes," adds "wireless earbuds" to the cart, and quickly closes the "laptop backpack" page. However, the user's device is shared by two family members, so some actions may not belong to this user. We form a neutrosophic set B = "items the current user intends to buy soon" on X:

$$\begin{aligned}
T_B(\text{running shoes}) &= 0.7 && \text{(multiple views + size filter)},\\
I_B(\text{running shoes}) &= 0.15 && \text{(not yet added to cart)},\\
F_B(\text{running shoes}) &= 0.1; &&\\
T_B(\text{wireless earbuds}) &= 0.6 && \text{(added to cart)},\\
I_B(\text{wireless earbuds}) &= 0.25 && \text{(could be another family member)},\\
F_B(\text{wireless earbuds}) &= 0.05; &&\\
T_B(\text{laptop backpack}) &= 0.2 && \text{(short page visit)},\\
I_B(\text{laptop backpack}) &= 0.4 && \text{(uncertain user identity)},\\
F_B(\text{laptop backpack}) &= 0.35 && \text{(bounce suggests low interest)}.
\end{aligned}$$

This neutrosophic representation lets the recommendation engine promote high–truth items, monitor high–indeterminacy items, and suppress high–falsity items.

Example 1.6 (Smart–home occupancy and energy control)**.** Let a smart–home controller try to decide which rooms are effectively occupied to adjust air conditioning. Let

$$X = \{\text{living room},\ \text{kitchen},\ \text{study}\}.$$

Sensors provide: motion (sometimes delayed), door open/close, and user calendar (which can be wrong). Because sensors can misfire and occupants can stay still (reading, working), we model "currently occupied rooms" as a neutrosophic set C on X:

$$\begin{aligned}
T_C(\text{living room}) &= 0.85 && \text{(recent motion + TV on)},\\
I_C(\text{living room}) &= 0.1 && \text{(possible pet movement)},\\
F_C(\text{living room}) &= 0.05; &&\\
T_C(\text{kitchen}) &= 0.4 && \text{(door opened 3 minutes ago)},\\
I_C(\text{kitchen}) &= 0.35 && \text{(no motion since then; could be passing through)},\\
F_C(\text{kitchen}) &= 0.2; &&\\
T_C(\text{study}) &= 0.3 && \text{(calendar says "work from home")},\\
I_C(\text{study}) &= 0.45 && \text{(no motion, user may have left)},\\
F_C(\text{study}) &= 0.2. &&
\end{aligned}$$

The controller can then cool the living room (high T), keep the kitchen in standby (high I), and reduce energy in the study (nonnegligible F), while still explicitly tracking indeterminacy to revise decisions when new sensor data arrive.

1.2 Plithogenic Set with Some Examples

A plithogenic set [42–45] represents elements via membership driven by explicit attributes together with a contradiction mapping between attribute values.

Definition 1.7 (Plithogenic Set)**.** [42,46] Let S be a universe and $P \subseteq S$ a nonempty subset. A *plithogenic set* is a 5–tuple

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF),$$

with the following ingredients:

- v — a chosen attribute;
- Pv — the value domain of v;
- $pdf : P \times Pv \to [0,1]^s$ — the *degree of appurtenance* (DAF);[1]

[1] In the literature, DAF is modeled in several equivalent ways (e.g., powerset–valued or vector–valued). We adopt the standard $[0,1]^s$ form; see [47].

- $pCF : Pv \times Pv \to [0,1]^t$ — the *degree of contradiction* (DCF).

For every $a, b \in Pv$, the DCF satisfies

$$\text{reflexivity: } pCF(a,a) = 0, \qquad \text{symmetry: } pCF(a,b) = pCF(b,a).$$

Here $s, t \in \mathbb{N}$ are, respectively, the appurtenance and contradiction dimensions.

Example 1.8 (Hiring decision — plithogenic *intersection* of two panels)**.** Let S be the set of applicants and $P = \{\mathrm{C}^\star\} \subseteq S$ the candidate under review. Choose the attribute $v =$ "competency" with value domain $Pv = \{\text{technical}, \text{communication}, \text{culture_fit}\}$ and dominant value $v^\star = \text{technical}$. The degree of contradiction to $v^\star$ is

$$c(\text{technical}) = 0, \qquad c(\text{communication}) = 0.4, \qquad c(\text{culture_fit}) = 0.7,$$

with pCF symmetric on $Pv \times Pv$. Two interview panels A, B provide appurtenances

$$\begin{aligned} &pdf_A(\mathrm{C}^\star, \text{technical}) = 0.85, \quad pdf_A(\mathrm{C}^\star, \text{communication}) = 0.65, \quad pdf_A(\mathrm{C}^\star, \text{culture_fit}) = 0.50, \\ &pdf_B(\mathrm{C}^\star, \text{technical}) = 0.75, \quad pdf_B(\mathrm{C}^\star, \text{communication}) = 0.55, \quad pdf_B(\mathrm{C}^\star, \text{culture_fit}) = 0.60. \end{aligned}$$

Use product t-norm $u \otimes v = uv$ and probabilistic s-norm $u \oplus v = u + v - uv$. The plithogenic *intersection* for value $a \in Pv$ is the contradiction-weighted mixer

$$d_{A\cap B}(\mathrm{C}^\star, a) \;=\; (1 - c(a))\big(pdf_A \otimes pdf_B\big) \;+\; c(a)\big(pdf_A \oplus pdf_B\big).$$

Concretely,

$$\begin{aligned} d_{A\cap B}(\text{technical}) &= (1-0)(0.85 \cdot 0.75) + 0(0.85 \oplus 0.75) = 0.6375, \\ d_{A\cap B}(\text{communication}) &= (1-0.4)(0.65 \cdot 0.55) + 0.4\,(0.65 + 0.55 - 0.65 \cdot 0.55) \\ &= 0.6 \cdot 0.3575 + 0.4 \cdot 0.8425 = 0.5515, \\ d_{A\cap B}(\text{culture_fit}) &= (1-0.7)(0.50 \cdot 0.60) + 0.7\,(0.50 + 0.60 - 0.50 \cdot 0.60) \\ &= 0.3 \cdot 0.30 + 0.7 \cdot 0.80 = 0.6500. \end{aligned}$$

(Optional scalarization) With $w(\text{technical}) = 0.5$, $w(\text{communication}) = 0.3$, $w(\text{culture_fit}) = 0.2$,

$$\mu^\star_{A\cap B}(\mathrm{C}^\star) = 1 - \prod_{a \in Pv} \big(1 - w(a)\, d_{A\cap B}(\mathrm{C}^\star, a)\big) \approx 0.5054.$$

Example 1.9 (Clinical diagnosis — plithogenic *union* of two evidence sources)**.** Let S be patients and $P = \{\mathrm{P}\} \subseteq S$ the case studied. Choose $v =$ "evidence type" with $Pv = \{\text{lab}, \text{imaging}, \text{symptoms}\}$ and dominant $v^\star = \text{lab}$. Set contradiction degrees

$$c(\text{lab}) = 0, \qquad c(\text{imaging}) = 0.3, \qquad c(\text{symptoms}) = 0.6.$$

Two sources A (ER tests) and B (primary-care assessment) report degrees supporting the hypothesis "*has pneumonia now*":

$$\begin{aligned} &pdf_A(\mathrm{P}, \text{lab}) = 0.70, \quad pdf_A(\mathrm{P}, \text{imaging}) = 0.80, \quad pdf_A(\mathrm{P}, \text{symptoms}) = 0.50, \\ &pdf_B(\mathrm{P}, \text{lab}) = 0.60, \quad pdf_B(\mathrm{P}, \text{imaging}) = 0.65, \quad pdf_B(\mathrm{P}, \text{symptoms}) = 0.55. \end{aligned}$$

Using the same norms as above, the plithogenic *union* for value a is

$$d_{A\cup B}(\mathrm{P}, a) \;=\; (1 - c(a))\big(pdf_A \oplus pdf_B\big) \;+\; c(a)\big(pdf_A \otimes pdf_B\big).$$

Numerical results:

$$\begin{aligned} d_{A\cup B}(\text{lab}) &= (1-0)(0.70 \oplus 0.60) = 0.70 + 0.60 - 0.42 = 0.8800, \\ d_{A\cup B}(\text{imaging}) &= (1-0.3)(0.80 \oplus 0.65) + 0.3\,(0.80 \cdot 0.65) \\ &= 0.7 \cdot 0.9300 + 0.3 \cdot 0.5200 = 0.8070, \\ d_{A\cup B}(\text{symptoms}) &= (1-0.6)(0.50 \oplus 0.55) + 0.6\,(0.50 \cdot 0.55) \\ &= 0.4 \cdot 0.7750 + 0.6 \cdot 0.2750 = 0.4750. \end{aligned}$$

(Optional scalarization) With $w(\text{lab}) = 0.5$, $w(\text{imaging}) = 0.3$, $w(\text{symptoms}) = 0.2$,

$$\mu^\star_{A\cup B}(\mathrm{P}) = 1 - \prod_{a \in Pv} \big(1 - w(a)\, d_{A\cup B}(\mathrm{P}, a)\big) \approx 0.616.$$

This yields an aggregated plithogenic support for the diagnosis, while the contradiction weights temper contributions away from the dominant (lab) evidence.

Chapter 2: Review and Result in Neutrosociology

In this section, we discuss Sociology and Neutrosociology.

2.1 Neutrosociology

Sociology studies social life, institutions, relationships, and change, using empirical methods to understand patterns, structures, power, culture, inequality, and conflict [48–51]. Neutrosociology applies neutrosophic logic to society, modeling truth, indeterminacy, and falsehood in social data, attributes, interactions, and evolution under uncertainty [52–56].

Definition 2.1 (Neutrosociology)**.** [52,53] Neutrosociology is the neutrosophic scientific study of society; that is, it studies sociological phenomena using neutrosophic methods and tools that explicitly account for indeterminacy. Concretely, social data and statements are modeled by neutrosophic components.

Formally, let X be a universe of social entities (individuals, groups, institutions). For any sociological attribute or statement A, we assign a single–valued neutrosophic appraisal

$$(T_A, I_A, F_A) : X \to [0,1]^3,$$

where for each $x \in X$:

$$T_A(x) \text{ is the degree of truth/membership of } x \text{ in } A,$$

$$I_A(x) \text{ is the degree of indeterminacy (neutral/unknown) of } x \text{ in } A,$$

$$F_A(x) \text{ is the degree of falsehood/nonmembership of } x \text{ in } A.$$

The three components are independent and (in the single–valued setting) satisfy

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

This triplet representation (T, I, F) enables quantitative analysis of vagueness, contradiction, and incomplete information in sociological research.

Example 2.2 (Public Health Compliance under Uncertain Evidence)**.** Let the universe be four residents

$$X = \{\text{Alice, Bob, Chen, Dana}\}.$$

Consider the sociological attribute

$$A = \text{“Complies with the Q4 booster vaccination campaign.”}$$

Using surveys (truth), conflicting records (falsehood), and missing data (indeterminacy), assign single–valued neutrosophic appraisals with sampling weights w (normalized to $\sum w = 1$):

x	Alice	Bob	Chen	Dana
$T_A(x)$	0.90	0.60	0.40	0.20
$I_A(x)$	0.05	0.25	0.30	0.40
$F_A(x)$	0.05	0.20	0.40	0.50
$w(x)$	0.40	0.30	0.20	0.10

Aggregate to a group–level appraisal using the standard neutrosophic accumulators:

$$T_A^{\text{grp}} = 1 - \prod_{x \in X} \big(1 - w(x)\, T_A(x)\big), \quad I_A^{\text{grp}} = \sum_{x \in X} w(x)\, I_A(x), \quad F_A^{\text{grp}} = \prod_{x \in X} \big(F_A(x)\big)^{w(x)}.$$

Numerical evaluation (step by step):

$$\begin{aligned} T_A^{\text{grp}} &= 1 - \big(1 - 0.4 \cdot 0.90\big)\big(1 - 0.3 \cdot 0.60\big)\big(1 - 0.2 \cdot 0.40\big)\big(1 - 0.1 \cdot 0.20\big) \\ &= 1 - (0.64)(0.82)(0.92)(0.98) = 1 - 0.47315968 = 0.52684032 \approx 0.527, \\ I_A^{\text{grp}} &= 0.4 \cdot 0.05 + 0.3 \cdot 0.25 + 0.2 \cdot 0.30 + 0.1 \cdot 0.40 = 0.020 + 0.075 + 0.060 + 0.040 = 0.195, \\ F_A^{\text{grp}} &= (0.05)^{0.4}(0.20)^{0.3}(0.40)^{0.2}(0.50)^{0.1} \\ &= \exp\big(0.4 \ln 0.05 + 0.3 \ln 0.20 + 0.2 \ln 0.40 + 0.1 \ln 0.50\big) \\ &= \exp(-1.1983 - 0.4828 - 0.1833 - 0.0693) = \exp(-1.9337) \approx 0.145. \end{aligned}$$

Thus the group–level neutrosophic assessment of “campaign compliance” is $(T, I, F) \approx (0.527,\ 0.195,\ 0.145)$.

Example 2.3 (Public Opinion on Urban Congestion Pricing)**.** Let the universe be five citizens

$$X = \{x_1, \dots, x_5\}, \qquad w(x_i) = 0.2 \ \text{(equal weights)}.$$

Consider the statement

$$A = \text{“Supports implementing congestion pricing downtown.”}$$

From interviews (truth), contradictory social posts (falsehood), and undecided responses (indeterminacy), set

x	x_1	x_2	x_3	x_4	x_5
$T_A(x)$	0.70	0.60	0.40	0.30	0.20
$I_A(x)$	0.10	0.20	0.40	0.30	0.20
$F_A(x)$	0.20	0.30	0.40	0.60	0.70

Group–level aggregation (same operators as Example 1):

$$T_A^{\text{grp}} = 1 - \prod_{i=1}^{5} \big(1 - 0.2\, T_A(x_i)\big), \quad I_A^{\text{grp}} = \tfrac{1}{5} \sum_{i=1}^{5} I_A(x_i), \quad F_A^{\text{grp}} = \prod_{i=1}^{5} \big(F_A(x_i)\big)^{0.2}.$$

Numerical evaluation:

$$\begin{aligned} T_A^{\text{grp}} &= 1 - (0.86)(0.88)(0.92)(0.94)(0.96) \\ &= 1 - 0.6273375744 \approx 0.373, \\ I_A^{\text{grp}} &= 0.2 \cdot (0.10 + 0.20 + 0.40 + 0.30 + 0.20) = 0.2 \cdot 1.20 = 0.24, \\ F_A^{\text{grp}} &= (0.20 \cdot 0.30 \cdot 0.40 \cdot 0.60 \cdot 0.70)^{0.2} = \exp\Big(0.2 \sum_{i=1}^{5} \ln F_A(x_i)\Big) \\ &= \exp\big(0.2 \cdot (-4.5972)\big) = \exp(-0.9194) \approx 0.399. \end{aligned}$$

Therefore the population–level appraisal of “policy support” is $(T, I, F) \approx (0.373,\ 0.24,\ 0.399)$.

2.2 Neutrosophic Social Evolution

Social evolution studies how societies change over time via adaptation, innovation, diffusion, conflict, cooperation, selection, environmental pressures, and institutional transformation [57–59]. Neutrosophic Social Evolution models societal change using triplet degrees: truth, indeterminacy, falsehood; semigroup dynamics updating attributes under evidence, interactions, uncertainty [60, 61].

Definition 2.4 (Neutrosophic Social Evolution)**.** (cf. [60, 61]) Fix a nonempty universe X of social entities (individuals, groups, institutions) and a set $\mathcal{A}$ of sociological attributes or statements. For each $A \in \mathcal{A}$ and time t in an index set $\mathbb{T}$ (discrete or continuous), a *neutrosophic social state* is a mapping

$$N_A(\cdot\,, t) : X \longrightarrow [0,1]^3, \qquad x \longmapsto \big(T_A(x,t),\ I_A(x,t),\ F_A(x,t)\big),$$

where T_A (truth/membership), I_A (indeterminacy/unknown), and F_A (falsehood/nonmembership) are independent components.

A *Neutrosophic Social Evolution (NSE)* is a family of evolution operators

$$\big(\Phi_\Delta\big)_{\Delta \in \mathbb{T}_+} : \mathcal{S} \times \mathcal{U} \longrightarrow \mathcal{S}, \quad \mathcal{S} := \big([0,1]^3\big)^{X \times \mathcal{A}},$$

such that the global state $N(\cdot, t) := \{N_A(\cdot, t)\}_{A \in \mathcal{A}}$ evolves by

$$N(\cdot, t + \Delta) \ = \ \Phi_\Delta\big(N(\cdot, t),\, U(t)\big),$$

where $U(t) \in \mathcal{U}$ encodes exogenous drivers (evidence, interactions, policies, shocks). The operators satisfy the semigroup axioms

$$\Phi_0 = \mathrm{id}, \qquad \Phi_{\Delta_1 + \Delta_2} = \Phi_{\Delta_2} \circ \Phi_{\Delta_1} \quad (\Delta_1, \Delta_2 \in \mathbb{T}_+),$$

and are *componentwise bounded* (images remain in $[0,1]^3$) and *isomorphism-invariant* (renaming of X preserves trajectories). Intuitively, NSE formalizes how the triplet (T, I, F) attached to social claims/attributes changes over time under information flow and interaction, thereby quantifying reinforcement (in T or F) and resolution or accumulation of uncertainty (in I).

Example 2.5 (Digital ID Support under an Information Campaign (Two Steps))**.** Let $X = \{\text{Aiko, Ben, Chen}\}$ and fix the statement $A =$ "Supports the new digital ID system." Use the probabilistic sum $a \oplus b := a + b - ab$ and the product t-norm $a \otimes b := ab$. Parameters: $\alpha = 0.6$ (pro-evidence gain), $\beta = 0.5$ (contra-evidence gain), $\gamma = 0.4$ (uncertainty decay so $1 - \gamma = 0.6$). The discrete-time NSE update is

$$\begin{aligned} T_A(x, t{+}1) &= T_A(x,t) \ \oplus \ \alpha\, e^{+}_{x,A}(t), \\ F_A(x, t{+}1) &= F_A(x,t) \ \oplus \ \beta\, e^{-}_{x,A}(t), \\ I_A(x, t{+}1) &= \big(I_A(x,t) \otimes (1-\gamma)\big) \ \oplus \ u_{x,A}(t). \end{aligned}$$

Initial state at $t = 0$:

x	$T_A(x,0)$	$I_A(x,0)$	$F_A(x,0)$
Aiko	0.30	0.50	0.20
Ben	0.50	0.30	0.30
Chen	0.20	0.40	0.40

Inputs at $t = 0$ (campaign starts): (e^+, e^-, u)

$$\text{Aiko} : (0.7, 0.2, 0.3), \quad \text{Ben} : (0.5, 0.4, 0.2), \quad \text{Chen} : (0.4, 0.5, 0.3).$$

Step $t = 0 \to 1$ (all arithmetic shown):

$$\begin{aligned} \text{Aiko}: \ T_1 &= 0.30 \oplus (0.6 \cdot 0.7{=}0.42) = 0.30 + 0.42 - 0.30 \cdot 0.42 = 0.594, \\ F_1 &= 0.20 \oplus (0.5 \cdot 0.2{=}0.10) = 0.20 + 0.10 - 0.02 = 0.28, \\ I_1 &= (0.50 \cdot 0.60{=}0.30) \oplus 0.30 = 0.30 + 0.30 - 0.09 = 0.51; \\ \text{Ben}: \ T_1 &= 0.50 \oplus (0.6 \cdot 0.5{=}0.30) = 0.50 + 0.30 - 0.15 = 0.65, \\ F_1 &= 0.30 \oplus (0.5 \cdot 0.4{=}0.20) = 0.30 + 0.20 - 0.06 = 0.44, \\ I_1 &= (0.30 \cdot 0.60{=}0.18) \oplus 0.20 = 0.18 + 0.20 - 0.036 = 0.344; \\ \text{Chen}: \ T_1 &= 0.20 \oplus (0.6 \cdot 0.4{=}0.24) = 0.20 + 0.24 - 0.048 = 0.392, \\ F_1 &= 0.40 \oplus (0.5 \cdot 0.5{=}0.25) = 0.40 + 0.25 - 0.10 = 0.55, \\ I_1 &= (0.40 \cdot 0.60{=}0.24) \oplus 0.30 = 0.24 + 0.30 - 0.072 = 0.468. \end{aligned}$$

Inputs at $t = 1$ (privacy safeguards announced):

$$\text{Aiko} : (e^+, e^-, u) = (0.8, 0.1, 0.1), \quad \text{Ben} : (0.7, 0.2, 0.15), \quad \text{Chen} : (0.6, 0.3, 0.2).$$

Step $t = 1 \to 2$:

$$\begin{aligned} \text{Aiko}: \ T_2 &= 0.594 \oplus (0.6 \cdot 0.8{=}0.48) = 0.594 + 0.48 - 0.594 \cdot 0.48 \\ &= 1.074 - 0.28512 = 0.78888, \\ F_2 &= 0.28 \oplus (0.5 \cdot 0.1{=}0.05) = 0.28 + 0.05 - 0.014 = 0.316, \\ I_2 &= (0.51 \cdot 0.60{=}0.306) \oplus 0.10 = 0.306 + 0.10 - 0.0306 = 0.3754; \\ \text{Ben}: \ T_2 &= 0.65 \oplus (0.6 \cdot 0.7{=}0.42) = 0.65 + 0.42 - 0.273 = 0.797, \\ F_2 &= 0.44 \oplus (0.5 \cdot 0.2{=}0.10) = 0.44 + 0.10 - 0.044 = 0.496, \\ I_2 &= (0.344 \cdot 0.60{=}0.2064) \oplus 0.15 = 0.2064 + 0.15 - 0.03096 = 0.32544; \\ \text{Chen}: \ T_2 &= 0.392 \oplus (0.6 \cdot 0.6{=}0.36) = 0.392 + 0.36 - 0.14112 = 0.61088, \\ F_2 &= 0.55 \oplus (0.5 \cdot 0.3{=}0.15) = 0.55 + 0.15 - 0.0825 = 0.6175, \\ I_2 &= (0.468 \cdot 0.60{=}0.2808) \oplus 0.20 = 0.2808 + 0.20 - 0.05616 = 0.42464. \end{aligned}$$

Interpretation: campaign reinforcement lifts T (Aiko, Ben markedly), counter-evidence shrinks F, and I declines as details clarify.

Example 2.6 (Mask Usage on Trains across Policy Phases (Two Steps))**.** Let $X = \{\text{Emi, Farah, Gopal}\}$ and $A =$ "Wears a mask on trains." Use $a \oplus b = a + b - ab$, $a \otimes b = ab$. Parameters: $\alpha = 0.5$, $\beta = 0.6$, $\gamma = 0.3$ (so $1 - \gamma = 0.7$).

Initial state at $t = 0$:

x	$T_A(x,0)$	$I_A(x,0)$	$F_A(x,0)$
Emi	0.60	0.20	0.20
Farah	0.40	0.30	0.30
Gopal	0.30	0.40	0.30

Inputs at $t = 0$ (case surge):

$$\text{Emi} : (e^+, e^-, u) = (0.6, 0.1, 0.2),\ \text{Farah} : (0.7, 0.1, 0.2),\ \text{Gopal} : (0.8, 0.1, 0.3).$$

Step $t = 0 \to 1$:

$$\begin{aligned}
\text{Emi} :\ T_1 &= 0.60 \oplus (0.5 \cdot 0.6{=}0.30) = 0.60 + 0.30 - 0.18 = 0.72,\\
F_1 &= 0.20 \oplus (0.6 \cdot 0.1{=}0.06) = 0.20 + 0.06 - 0.012 = 0.248,\\
I_1 &= (0.20 \cdot 0.70{=}0.14) \oplus 0.20 = 0.14 + 0.20 - 0.028 = 0.312;\\
\text{Farah} :\ T_1 &= 0.40 \oplus (0.5 \cdot 0.7{=}0.35) = 0.40 + 0.35 - 0.14 = 0.61,\\
F_1 &= 0.30 \oplus 0.06 = 0.30 + 0.06 - 0.018 = 0.342,\\
I_1 &= (0.30 \cdot 0.70{=}0.21) \oplus 0.20 = 0.21 + 0.20 - 0.042 = 0.368;\\
\text{Gopal} :\ T_1 &= 0.30 \oplus (0.5 \cdot 0.8{=}0.40) = 0.30 + 0.40 - 0.12 = 0.58,\\
F_1 &= 0.30 \oplus 0.06 = 0.30 + 0.06 - 0.018 = 0.342,\\
I_1 &= (0.40 \cdot 0.70{=}0.28) \oplus 0.30 = 0.28 + 0.30 - 0.084 = 0.496.
\end{aligned}$$

Inputs at $t = 1$ (heat wave; mandate lifted):

$$\text{Emi} : (e^+, e^-, u) = (0.2, 0.6, 0.1),\ \text{Farah} : (0.2, 0.5, 0.2),\ \text{Gopal} : (0.2, 0.4, 0.2).$$

Step $t = 1 \to 2$:

$$\begin{aligned}
\text{Emi} :\ T_2 &= 0.72 \oplus (0.5 \cdot 0.2{=}0.10) = 0.72 + 0.10 - 0.072 = 0.748,\\
F_2 &= 0.248 \oplus (0.6 \cdot 0.6{=}0.36) = 0.248 + 0.36 - 0.248 \cdot 0.36\\
&= 0.608 - 0.08928 = 0.51872,\\
I_2 &= (0.312 \cdot 0.70{=}0.2184) \oplus 0.10 = 0.2184 + 0.10 - 0.02184 = 0.29656;\\
\text{Farah} :\ T_2 &= 0.61 \oplus 0.10 = 0.61 + 0.10 - 0.061 = 0.649,\\
F_2 &= 0.342 \oplus (0.6 \cdot 0.5{=}0.30) = 0.342 + 0.30 - 0.1026 = 0.5394,\\
I_2 &= (0.368 \cdot 0.70{=}0.2576) \oplus 0.20 = 0.2576 + 0.20 - 0.05152 = 0.40608;\\
\text{Gopal} :\ T_2 &= 0.58 \oplus 0.10 = 0.58 + 0.10 - 0.058 = 0.622,\\
F_2 &= 0.342 \oplus (0.6 \cdot 0.4{=}0.24) = 0.342 + 0.24 - 0.08208 = 0.49992,\\
I_2 &= (0.496 \cdot 0.70{=}0.3472) \oplus 0.20 = 0.3472 + 0.20 - 0.06944 = 0.47776.
\end{aligned}$$

Interpretation: initial surge raises T across agents; later policy relaxation raises F and partially resolves/accumulates I, illustrating NSE's capacity to track reinforcement and uncertainty dynamics over time.

2.3 Neutrosophic Sociogram

A sociogram is a visual diagram mapping social relationships, showing who chooses, rejects, or connects within groups for sociometric analysis [62–64]. A neutrosophic sociogram represents social ties with truth, indeterminacy, and falsity degrees, distinguishing actual, uncertain, and potential connections within networks [65, 66]. Furthermore, as a further extended concept, the Plithogenic Sociogram is also known [47, 67, 68].

Definition 2.7 (Neutrosophic Sociogram)**.** [66] Let $S = \{s_1, \ldots, s_n\}$ be a finite social group and let $Q = \{Q_1, \ldots, Q_m\}$ be a set of sociometric questions (e.g. "with whom would you like to study?", "with whom would you do a project?", "with whom are you unsure?"). For each $s_i \in S$ and each question Q_j we collect two disjoint sets

$$S_{ij} \subseteq S, \qquad O_{ij} \subseteq S, \qquad S_{ij} \cap O_{ij} = \varnothing,$$

where S_{ij} is the set of members that s_i *positively* chooses for Q_j, and O_{ij} is the set of members about whom s_i is *indeterminate* (not sure, no experience, or potentially future link) for Q_j. Let $\Omega = \{\omega_1, \ldots, \omega_m\}$ be nonnegative weights with $\sum_{j=1}^{m} \omega_j = 1$.

For each question Q_j define the preference matrix $R^{(j)} = (r_{kl}^{(j)})_{k,l=1}^{n}$ by

$$r_{kl}^{(j)} = \begin{cases} 1, & \text{if } s_l \in S_{kj} \text{ or } k = l,\\ 0, & \text{otherwise,} \end{cases}$$

and the *fuzzy preference matrix*

$$F = \sum_{j=1}^{m} \omega_j R^{(j)} = (f_{kl})_{k,l=1}^{n}.$$

Symmetrizing, define the *fuzzy amicable degree*

$$g_{kl} = \frac{2}{\frac{1}{f_{kl}} + \frac{1}{f_{lk}}}$$

with the usual conventions for 0 and ∞. Analogously, for the indeterminate selections define $T^{(j)} = (t_{kl}^{(j)})$ by

$$t_{kl}^{(j)} = \begin{cases} 1, & \text{if } s_l \in S_{kj} \text{ or } s_l \in O_{kj} \text{ or } k = l, \\ 0, & \text{otherwise,} \end{cases}$$

and the aggregated matrix

$$T = \sum_{j=1}^{m} \omega_j T^{(j)} = (t_{kl})_{k,l=1}^{n},$$

then the *neutrosophic amicable degree* is

$$u_{kl} = \frac{2}{\frac{1}{t_{kl}} + \frac{1}{t_{lk}}}.$$

For every unordered pair $\{s_k, s_l\}$, we define the *neutrosophic link interval*

$$I_{kl} = [g_{kl},\, u_{kl}],$$

where g_{kl} measures the *current* (confirmed) tie and u_{kl} measures the *potential* tie that may be realized in the future. The *Neutrosophic Sociogram* is the undirected neutrosophic graph on S such that

- a solid edge s_k—s_l is drawn if $g_{kl} > 0$ (confirmed preference),
- a dashed edge s_k—s_l is drawn if $g_{kl} = 0$ but $u_{kl} > 0$ (indeterminate/potential link),
- each edge is labelled by the interval $I_{kl} = [g_{kl}, u_{kl}]$.

This structure distinguishes absence of relation from undecided/potential relation and therefore refines classical and fuzzy sociograms.

Example 2.8 (Small work team)**.** Consider a project team $S = \{a, b, c, d\}$ in a company. The manager asks two questions:

Q_1 : "With whom would you like to prepare the monthly report?",
Q_2 : "With whom are you not sure yet, but could possibly collaborate in the future?".

The answers are:

member	Q_1 (preferred)	Q_2 (not sure / potential)
a	$\{b, c\}$	$\{d\}$
b	$\{a\}$	$\{c\}$
c	$\{a\}$	$\{b, d\}$
d	$\varnothing$	$\{a, c\}$

Take equal weights $\omega_1 = \omega_2 = \frac{1}{2}$. Then the fuzzy preference matrix $F = (f_{xy})$ has

$$f_{ab} = \tfrac{1}{2},\ f_{ac} = \tfrac{1}{2},\ f_{ba} = \tfrac{1}{2},\ f_{ca} = \tfrac{1}{2},\ f_{xx} = 1 \text{ for all } x,$$

and $f_{xy} = 0$ otherwise. Symmetrizing, we get (for $x \neq y$)

$$g_{ab} = \frac{2}{\frac{1}{1/2} + \frac{1}{1/2}} = \frac{2}{2+2} = \tfrac{1}{2}, \quad g_{ac} = \tfrac{1}{2}, \quad g_{bc} = 0, \quad g_{ad} = g_{bd} = g_{cd} = 0.$$

Next, for the enlarged (preferred or potential) selections we obtain

$$t_{ad} = \tfrac{1}{2},\ t_{bd} = 0,\ t_{cd} = \tfrac{1}{2},\ t_{da} = \tfrac{1}{2},\ t_{dc} = \tfrac{1}{2},$$

and (because c listed b in Q_2 and b listed c in Q_2) we have

$$t_{bc} = \tfrac{1}{2}, \quad t_{cb} = \tfrac{1}{2}.$$

Hence the neutrosophic amicable degrees are

$$u_{bc} = \frac{2}{\frac{1}{1/2} + \frac{1}{1/2}} = \tfrac{1}{2}, \quad u_{ad} = \frac{2}{\frac{1}{1/2} + \frac{1}{1/2}} = \tfrac{1}{2}, \quad u_{cd} = \tfrac{1}{2},$$

while $u_{ab} = \frac{1}{2}$ and $u_{ac} = \frac{1}{2}$ coincide with their fuzzy parts.

Therefore the neutrosophic sociogram is:

- solid edges: a—b with label $[0.5, 0.5]$, a—c with label $[0.5, 0.5]$;
- dashed edges (purely potential): b—c with label $[0, 0.5]$, a—d with label $[0, 0.5]$, c—d with label $[0, 0.5]$;
- d has no solid edge but has neutrosophic/potential ties to a and c, so the manager can plan joint tasks to activate them.

This tiny example shows exactly what the neutrosophic sociogram adds to a fuzzy sociogram: it makes visible those relations that are neither present nor absent, but undecided and improvable.

2.4 Neutrosophic Social Change

Neutrosophic social change describes societal transformations by assigning truth, indeterminacy, and falsity degrees to each targeted social attribute over time [61].

Definition 2.9 (Neutrosophic Social Change)**.** (cf. [61]) A *neutrosophic social change* is the transformation of a social system from an initial social configuration $S(t_0)$ to a later configuration $S(t_1)$, described for each relevant social attribute $a \in \mathcal{A}$ (e.g. education access, gender equality, technological adoption, trust, participation) by a neutrosophic triplet

$$C(a) \;=\; \big(T_a,\, I_a,\, F_a\big), \qquad T_a, I_a, F_a \subseteq [0, 1],$$

where

- T_a is the degree to which the change in attribute a is effectively achieved/realized (truth– or membership–degree of change),
- I_a is the degree to which the change in a is indeterminate, unclear, conflicted, or only partially observable (indeterminacy–degree),
- F_a is the degree to which the intended change in a is not achieved, is resisted, or is contradicted in practice (falsehood– or nonmembership–degree).

The collection

$$\mathrm{NSC}(S; t_0 \to t_1) \;=\; \{\, (a; T_a, I_a, F_a) \mid a \in \mathcal{A} \,\}$$

is called the *neutrosophic social change profile*. It generalizes classical and fuzzy views on social change by keeping, for every attribute, the simultaneously present success, failure, and indeterminacy components that typically occur in real societies.

Example 2.10 (Technological diffusion with contradictory effects)**.** Consider a mid-size city that, between 2025 and 2027, introduces a national e-government portal, public Wi-Fi in libraries, and a digital-identification system. Let

$$\mathcal{A} = \{\text{service-access}, \text{digital-trust}, \text{digital-divide}\}.$$

After two years:

- For service-access, surveys and usage logs show that about 70% of the population actively uses the platform, another 10% of the data is contradictory (some agencies report use, others do not), and roughly 20% of citizens do not or cannot use it. We record
$$C(\text{service-access}) = (0.70,\, 0.10,\, 0.20).$$
- For digital-trust, political analysts give split assessments: part of the population appreciates transparency, but there are rumors about data misuse from unverified sources; thus
$$C(\text{digital-trust}) = (0.45,\, 0.35,\, 0.25),$$
meaning that trust increased almost by half, yet more than one third is indeterminate.

- For digital-divide, field studies show that elderly and low-income groups benefited less; some NGOs report no change, while education offices report slight improvement. Hence

$$C(\text{digital-divide}) = (0.30,\ 0.40,\ 0.35).$$

The profile

$$\text{NSC}(S; 2025 \to 2027) = \{(\text{service-access}; 0.70, 0.10, 0.20), (\text{digital-trust}; 0.45, 0.35, 0.25), (\text{digital-divide}; 0.30, 0.40, 0.35)\}$$

shows that the same social change can be simultaneously successful, debated/uncertain, and resisted, which is exactly what the neutrosophic modeling aims to capture.

2.5 Neutrosophic Ideal Social Group

A neutrosophic ideal social group maximizes cooperative truths, records unavoidable indeterminacies, and minimizes falsities across members' positive relations and interactions [52].

Definition 2.11 (Neutrosophic Ideal Social Group)**.** [52] A *neutrosophic ideal social group* is a finite social group

$$G = \{g_1, \dots, g_n\}$$

together with a family of neutrosophic evaluations

$$E = \left\{ e_j(g_i) = (T_{ij}, I_{ij}, F_{ij}) \in [0,1]^3 \;\middle|\; 1 \le i \le n,\ 1 \le j \le m \right\}$$

on m positive sociological attributes

$$\mathcal{P} = \{\text{cooperation}, \text{reciprocity}, \text{norm-compliance}, \text{mutual-support}, \dots\},$$

such that

1. for every member g_i and every positive attribute $p_j \in \mathcal{P}$ we have

$$T_{ij} \text{ is high (close to 1)}, \quad F_{ij} \text{ is low (close to 0)},$$

2. the indeterminacy I_{ij} is either small (the group is well known, stable) or explicitly recorded (the group keeps track of unclear relations),

3. the group also records neutrosophic links among members, e.g. a neutrosophic sociogram

$$\sigma(g_k, g_\ell) = (T_{k\ell}, I_{k\ell}, F_{k\ell}),$$

where $T_{k\ell}$ is the strength of the social tie, $I_{k\ell}$ is the doubt/potentiality of the tie, and $F_{k\ell}$ is the refusal or conflict between g_k and g_ℓ,

4. and for the group as a whole there exists an aggregated triplet

$$\text{NI}(G) = (T_G, I_G, F_G)$$

with T_G maximal among comparable groups, F_G minimal, and I_G controlled, expressing that the group is *ideally* cooperative, but still realistically allows for indeterminacy.

In words, a neutrosophic ideal social group is an *idealized* but *not* perfectly crisp group: it pushes the truth/membership of all positive social rules near 1, pushes the falsehood/nonmembership of those rules near 0, yet keeps an explicit place for uncertainty and contradictory assessments, because real groups always contain such parts.

Example 2.12 (Research team with explicit uncertainty tracking)**.** Let $G = \{R_1, R_2, R_3, R_4\}$ be a university research team working on neutrosophic sociograms. The leader R_1 collects, every semester, the following neutrosophic evaluations on three attributes

$$\mathcal{P} = \{\text{task-cooperation}, \text{information-sharing}, \text{conflict-management}\}.$$

Suppose the aggregated evaluations (averaged over members and over the last two projects) are

$$\begin{aligned} (\text{task-cooperation}) &= (0.90,\ 0.05,\ 0.05), \\ (\text{information-sharing}) &= (0.85,\ 0.10,\ 0.08), \\ (\text{conflict-management}) &= (0.78,\ 0.15,\ 0.12). \end{aligned}$$

The second line shows that information is shared very well, but there is 10% indeterminacy because two members joined only recently and their behavior is not yet fully observed. The conflict-management attribute has $(0.78, 0.15, 0.12)$, i.e., the group is mostly positive, but there are both unclear cases (ongoing discussions) and a small explicit conflict rate. On the sociogram level, we also record

$$\sigma(R_2, R_4) = (0.60, 0.30, 0.10),$$

meaning that R_2 and R_4 usually collaborate, 30% of their relation is not yet known (new common topic), and only 10% is negative. Because T-components are high on all positive attributes, F-components are low, and indeterminacy is acknowledged rather than ignored, G can be regarded as a *neutrosophic ideal social group* in the sense of Definition 2.11.

2.6 Neutrosophic Microsociology

Microsociology studies everyday face-to-face interactions to reveal how meanings, identities, and social order emerge from routine situations in small groups (cf. [69–71]). Neutrosophic microsociology models individual interactions using triplets of truth, indeterminacy, and falsity to capture ambiguous, conflicting, or partial realities precisely [52].

Definition 2.13 (Neutrosophic Microsociology)**.** [52] Neutrosophic Microsociology is the small–scale, interaction–level study of social life in which each observed act, utterance, role–performance, or dyadic/triadic encounter is described by a neutrosophic triplet

$$(T, I, F) \in [0, 1]^3,$$

where

- T (truth/membership) measures how far the interaction realizes the intended or normatively expected social meaning (e.g. "respectful greeting", "effective help", "cooperative turn–taking");
- I (indeterminacy) measures ambiguity, partial observation, conflicting accounts, or contextual under–specification in the same interaction;
- F (falsity/nonmembership) measures how far the interaction fails to realize, or even contradicts, the expected meaning (e.g. "refusal", "hidden resistance", "cold response").

Formally, fix

$$A = \text{finite set of social actors}, \qquad E = \text{multiset of observed micro–events}, \qquad C = \text{set of interactional contexts}.$$

A neutrosophic microsociological observation is a map

$$\mathcal{M} : A \times A \times E \times C \longrightarrow [0, 1]^3, \qquad (a, b, e, c) \longmapsto \big(T_{ab}^{(e,c)},\, I_{ab}^{(e,c)},\, F_{ab}^{(e,c)}\big),$$

with $0 \le T + I + F \le 3$. The pair $(A, \mathcal{M})$ is called a *neutrosophic microsociological structure*. It refines classical and fuzzy microsociology by keeping, for *every* single interaction, its simultaneous success, ambiguity, and failure components.

Example 2.14 (Nurse–patient greeting in an outpatient clinic)**.** Consider an outpatient clinic where the head nurse wants to monitor the *quality of micro–interactions* at the reception desk. Let

$$A = \{\text{Nurse}_1, \text{Nurse}_2, \text{Patient}_1, \text{Patient}_2\}, \quad E = \{\text{arrival}, \text{information request}, \text{checkout}\}, \quad C = \{\text{morning}, \text{afternoon}\}.$$

For the event "arrival" in the "morning" context, the expected script is: "nurse greets patient warmly, confirms name, and guides to waiting area." Observation (1): Nurse_1 greets Patient_1 with eye contact and guiding gesture, but the waiting area is crowded, so part of the instruction is unclear. We record

$$\mathcal{M}(\text{Nurse}_1, \text{Patient}_1, \text{arrival}, \text{morning}) = (T, I, F) = (0.82,\ 0.12,\ 0.10).$$

Here $T = 0.82$ is high because the greeting was appropriate; $I = 0.12$ captures the crowd–related uncertainty (the patient might not have heard all instructions); $F = 0.10$ captures a minor failure (the nurse forgot to tell the approximate waiting time).

Observation (2): Nurse_2 greets Patient_2 but is interrupted by a phone call and does not complete the guiding part. We record

$$\mathcal{M}(\text{Nurse}_2, \text{Patient}_2, \text{arrival}, \text{morning}) = (0.55,\ 0.28,\ 0.35),$$

meaning: the core of the greeting is present ($T = 0.55$), a significant part is ambiguous ($I = 0.28$) because the patient received only half of the information, and there is a nonnegligible failure ($F = 0.35$) due to the interruption.

If we aggregate over the morning arrivals for the two nurses by a simple neutrosophic average,

$$T_{\text{morning}} = \frac{0.82 + 0.55}{2} = 0.685, \quad I_{\text{morning}} = \frac{0.12 + 0.28}{2} = 0.20, \quad F_{\text{morning}} = \frac{0.10 + 0.35}{2} = 0.225.$$

Interpretation: at the *micro* level, the clinic's morning greetings are mostly successful ($T_{\text{morning}} \approx 0.69$), but a fifth of the behavior is ambiguous ($I_{\text{morning}} = 0.20$), and almost a quarter represents clear deviation or failure ($F_{\text{morning}} = 0.225$). This is exactly the kind of fine–grained description that neutrosophic microsociology is meant to provide: it does not force us to choose "good" or "bad," but keeps the three concurrent components of everyday interaction.

2.7 Neutrosophic Functionalism

Neutrosophic Functionalism formally analyzes social structures' functions via neutrosophic triplets, aggregating role contributions, uncertainty, and failures, with compositional, bounded operators [52].

Definition 2.15 (Neutrosophic Functionalism)**.** [52] Let U be a nonempty set of social units (individuals, groups) and let $\mathcal{S} \subseteq \mathcal{P}(U)$ be the family of social structures (e.g., roles, teams, institutions). Let $\mathcal{G}$ be a set of social functions (norms, services, goals). A *Neutrosophic Functionalist model* is a tuple

$$\mathfrak{NF} = \big(U, \mathcal{S}, \mathcal{G},\, r,\, w,\, \oplus,\, \mathsf{A}_I\big)$$

consisting of:

- a role–level neutrosophic appraisal

  $$r : U \times \mathcal{G} \to [0,1]^3, \qquad r(u,g) = \big(T_r(u,g),\, I_r(u,g),\, F_r(u,g)\big),$$

  where T_r (truth/membership), I_r (indeterminacy/unknown), and F_r (falsehood/nonmembership) quantify how unit u contributes to realizing function g;
- weights $w : U \to [0,1]$ encoding the salience/capacity of each unit;
- an s-norm $\oplus : [0,1]^2 \to [0,1]$ (commutative, associative, monotone, with neutral 0), used to accumulate truth;
- a monotone, commutative aggregator A_I on $[0,1]$ (e.g., weighted mean, max), used to pool indeterminacy.

From r we define the *structure–level* neutrosophic functional relation

$$N : \mathcal{S} \times \mathcal{G} \to [0,1]^3, \qquad N(S,g) = \big(T_{S,g},\, I_{S,g},\, F_{S,g}\big),$$

by the explicit formulas

$$T_{S,g} \;:=\; 1 - \prod_{u \in S} \big(1 - w(u)\, T_r(u,g)\big), \qquad I_{S,g} \;:=\; \mathsf{A}_I\big(\{\, w(u)\, I_r(u,g) : u \in S \,\}\big), \qquad F_{S,g} \;:=\; \prod_{u \in S} \big(F_r(u,g)\big)^{w(u)}.$$

These yield $0 \le T_{S,g}, I_{S,g}, F_{S,g} \le 1$ for all (S,g), without requiring $T_{S,g} + I_{S,g} + F_{S,g} = 1$.

The pair $\big(\mathcal{S}, N\big)$ is called a *neutrosophic functionalist account* of society: for each structure S and function g, the triplet $N(S,g)$ states, respectively, the degrees to which S realizes g, remains uncertain about g, or fails g.

Two canonical properties hold.

1. Monotonicity in membership: if $S \subseteq S'$ then $T_{S,g} \le T_{S',g}$ and $F_{S,g} \ge F_{S',g}$ for all $g \in \mathcal{G}$.
2. Compositionality (disjoint union): for $S_1 \cap S_2 = \varnothing$,

   $$T_{S_1 \cup S_2, g} \;=\; T_{S_1,g} \;\oplus\; T_{S_2,g}, \qquad I_{S_1 \cup S_2, g} \;=\; \mathsf{A}_I\big(I_{S_1,g}, I_{S_2,g}\big), \qquad F_{S_1 \cup S_2, g} \;=\; F_{S_1,g} \cdot F_{S_2,g}$$

   when $\oplus$ is the probabilistic sum $a \oplus b = a + b - ab$ and A_I is, e.g., a (weighted) mean.

For priorities $\mu : \mathcal{G} \to [0,1]$ and penalties $\alpha, \beta \in [0,1]$, a system–level viability score is

$$\mathrm{Viab}(S) := \sum_{g \in \mathcal{G}} \mu(g)\, \big(T_{S,g} - \alpha\, I_{S,g} - \beta\, F_{S,g}\big),$$

and a thresholded satisfaction relation at $(\tau, \iota, \varphi) \in [0,1]^3$ is

$$S \models_{(\tau,\iota,\varphi)} g \quad \Longleftrightarrow \quad T_{S,g} \ge \tau,\;\; I_{S,g} \le \iota,\;\; F_{S,g} \le \varphi.$$

Thus, *Neutrosophic Functionalism* is the view that social order and change are analyzed via the triplet-valued map N, which explicitly tracks truth, indeterminacy, and falsehood of functions across structures, with clear composition and decision rules.

Example 2.16 (Hospital ER Team Realizing "Rapid Triage within 10 Minutes")**.** Let the social units be

$$U = \{\text{Nurse } (N),\ \text{Physician } (P),\ \text{Registration } (R),\ \text{IT Support } (I)\},$$

and the function be $g =$ "Rapid triage within 10 minutes". Choose salience weights that sum to 1:

$$w(N) = 0.35, \quad w(P) = 0.40, \quad w(R) = 0.15, \quad w(I) = 0.10.$$

From logs/observations, the role–level neutrosophic appraisals $r(u, g) = (T_r, I_r, F_r)$ are

u	N	P	R	I
$T_r(u, g)$	0.85	0.75	0.70	0.60
$I_r(u, g)$	0.10	0.15	0.20	0.25
$F_r(u, g)$	0.08	0.18	0.22	0.30

Take $\oplus$ to be the probabilistic sum $a \oplus b = a + b - ab$ (for truth aggregation) and A_I the weighted mean $\sum_{u \in S} w(u)\, I_r(u, g)$ (weights sum to 1). Then by the model,

$$T_{S,g} = 1 - \prod_{u \in S} \big(1 - w(u)\, T_r(u, g)\big), \quad I_{S,g} = \sum_{u \in S} w(u)\, I_r(u, g), \quad F_{S,g} = \prod_{u \in S} \big(F_r(u, g)\big)^{w(u)}.$$

Numerical evaluation (all steps shown):

$$\begin{aligned} T_{S,g} &= 1 - \big(1 - 0.35 \cdot 0.85\big)\big(1 - 0.40 \cdot 0.75\big)\big(1 - 0.15 \cdot 0.70\big)\big(1 - 0.10 \cdot 0.60\big) \\ &= 1 - (0.7025)(0.7000)(0.8950)(0.9400) \\ &= 1 - 0.413709275 \;=\; 0.586290725, \end{aligned}$$

$$\begin{aligned} I_{S,g} &= 0.35 \cdot 0.10 + 0.40 \cdot 0.15 + 0.15 \cdot 0.20 + 0.10 \cdot 0.25 \\ &= 0.035 + 0.060 + 0.030 + 0.025 \;=\; 0.150, \end{aligned}$$

$$\begin{aligned} F_{S,g} &= (0.08)^{0.35}(0.18)^{0.40}(0.22)^{0.15}(0.30)^{0.10} \\ &= \exp\Big(\sum_u w(u) \ln F_r(u, g)\Big) = \exp(-1.9174408371) \;=\; 0.1469826337. \end{aligned}$$

Sanity check (compositionality). Let $S_1 = \{N, P\}$ (clinical core) and $S_2 = \{R, I\}$ (support). Compute

$$\begin{aligned} T_{S_1,g} &= 1 - (1 - 0.35 \cdot 0.85)(1 - 0.40 \cdot 0.75) = 0.50825, \\ T_{S_2,g} &= 1 - (1 - 0.15 \cdot 0.70)(1 - 0.10 \cdot 0.60) = 0.1587, \\ T_{S_1 \cup S_2,g} &= T_{S_1,g} \,\oplus\, T_{S_2,g} = 0.50825 + 0.1587 - 0.50825 \cdot 0.1587 = 0.586290725, \end{aligned}$$

and similarly $I_{S_1 \cup S_2,g} = I_{S_1,g} + I_{S_2,g} = 0.095 + 0.055 = 0.150$, $F_{S_1 \cup S_2,g} = F_{S_1,g} \cdot F_{S_2,g} = 0.1469826337$, matching the direct computation above.

Example 2.17 (IT Service Desk Realizing "Resolve Incidents within SLA")**.** An IT Service Desk is the central point that handles incidents, service requests, and user support across organizational IT operations [72, 73]. Let the units be Level-1 Agent A (A), Level-1 Agent B (B), Level-2 Engineer $(L2)$, and On-call Manager (M), with weights (capacity/salience) summing to 1:

$$w(A) = 0.30, \quad w(B) = 0.25, \quad w(L2) = 0.30, \quad w(M) = 0.15.$$

Observed role–level appraisals for the function $g =$ "Resolve within SLA":

u	A	B	$L2$	M
$T_r(u, g)$	0.70	0.60	0.80	0.65
$I_r(u, g)$	0.15	0.20	0.10	0.20
$F_r(u, g)$	0.20	0.25	0.15	0.20

Using the same operators as Example 1, the structure–level appraisal is

$$T_{S,g} = 1 - \prod_u (1 - w(u) T_r(u, g)), \quad I_{S,g} = \sum_u w(u) I_r(u, g), \quad F_{S,g} = \prod_u \big(F_r(u, g)\big)^{w(u)}.$$

Numerical evaluation:

$$\begin{aligned} T_{S,g} &= 1 - \big(1 - 0.30 \cdot 0.70\big)\big(1 - 0.25 \cdot 0.60\big)\big(1 - 0.30 \cdot 0.80\big)\big(1 - 0.15 \cdot 0.65\big) \\ &= 1 - (0.79)(0.85)(0.76)(0.9025) = 1 - 0.46058185 \;=\; 0.53941815, \end{aligned}$$

$$\begin{aligned} I_{S,g} &= 0.30 \cdot 0.15 + 0.25 \cdot 0.20 + 0.30 \cdot 0.10 + 0.15 \cdot 0.20 \\ &= 0.045 + 0.050 + 0.030 + 0.030 \;=\; 0.155, \end{aligned}$$

$$\begin{aligned} F_{S,g} &= (0.20)^{0.30}(0.25)^{0.25}(0.15)^{0.30}(0.20)^{0.15} \\ &= \exp\Big(0.30 \ln 0.20 + 0.25 \ln 0.25 + 0.30 \ln 0.15 + 0.15 \ln 0.20\Big) \\ &= \exp(-1.6399566463) \;=\; 0.1939884522. \end{aligned}$$

Disjoint decomposition check. Let $S_1 = \{A, B\}$ (frontline) and $S_2 = \{L2, M\}$ (escalation/management). Then

$$T_{S_1,g} = 1 - (1 - 0.30 \cdot 0.70)(1 - 0.25 \cdot 0.60) = 0.3285, \qquad I_{S_1,g} = 0.095, \qquad F_{S_1,g} = (0.20)^{0.30}(0.25)^{0.25} = 0.4363088286,$$
$$T_{S_2,g} = 1 - (1 - 0.30 \cdot 0.80)(1 - 0.15 \cdot 0.65) = 0.3141, \qquad I_{S_2,g} = 0.060, \qquad F_{S_2,g} = (0.15)^{0.30}(0.20)^{0.15} = 0.4446127136.$$

With $\oplus$ the probabilistic sum and A_I the (weight-consistent) sum,

$$\begin{aligned} T_{S_1 \cup S_2,g} &= T_{S_1,g} \ \oplus \ T_{S_2,g} = 0.3285 + 0.3141 - 0.3285 \cdot 0.3141 = 0.53941815, \\ I_{S_1 \cup S_2,g} &= I_{S_1,g} + I_{S_2,g} = 0.095 + 0.060 = 0.155, \\ F_{S_1 \cup S_2,g} &= F_{S_1,g} \cdot F_{S_2,g} = 0.1939884522, \end{aligned}$$

agreeing exactly with the direct structure computation.

(Optional viability score for a single g). For penalties $(\alpha, \beta) = (0.5, 0.7)$,

$$\mathrm{Viab}(S) = T_{S,g} - \alpha I_{S,g} - \beta F_{S,g} = \begin{cases} 0.5863 - 0.5 \cdot 0.150 - 0.7 \cdot 0.1470 \approx 0.4084 & \text{(ER team)}, \\ 0.5394 - 0.5 \cdot 0.155 - 0.7 \cdot 0.1940 \approx 0.3261 & \text{(Service desk)}. \end{cases}$$

These concrete calculations illustrate how Neutrosophic Functionalism aggregates role $\to$ structure evidence, quantifies uncertainty, and composes substructures while preserving bounds and the stated axioms.

Chapter 3: Review and Result in Psychology

In the field of Psychology, both Fuzzy Sets and Neutrosophic Sets have been applied from different perspectives, providing ways to represent uncertainty, vagueness, and indeterminacy in human cognition and behavior [74–76]. Building on this foundation, in this section we examine concepts such as *Neutropsyche*, which extends neutrosophic logic to psychological processes including memory, traits, and decision–making.

3.1 Neutropsyche

Psychology studies mind and behavior, investigating cognition, emotion, perception, development, personality, motivation, mental health, and social interactions through scientific methods [77–80]. Neutropsyche studies psychological entities using neutrosophic truth, indeterminacy, and falsehood, modeling beliefs, memories, traits, and contradictions quantitatively over time scales [81–84]. Moreover, a related concept known as Fuzzy Psychology is also recognized [85–87].

Definition 3.1 (Neutropsyche)**.** [81] Neutropsyche is the *neutrosophic psychological theory*: it studies psychological entities using the neutrosophic triad of opposites and their neutral $(\langle A\rangle, \langle \mathrm{neut}A\rangle, \langle \mathrm{anti}A\rangle)$. Formally, let $\mathcal{E}$ be a universe of psychological entities (attributes, statements, hypotheses, traits, memories). A *single-valued neutrosophic appraisal* on $\mathcal{E}$ is a map

$$\mathrm{NL} : \mathcal{E} \longrightarrow [0,1]^3, \qquad e \longmapsto \big(T(e),\, I(e),\, F(e)\big),$$

where $T(e)$ is the degree of truth / membership of e, $I(e)$ the degree of indeterminacy (neutrality), and $F(e)$ the degree of falsehood / nonmembership. In the single-valued setting,

$$T(e), I(e), F(e) \in [0,1], \qquad 0 \le T(e) + I(e) + F(e) \le 3,$$

and the components are mutually independent. Within psychology, this appraisal is used both for propositions (e.g., theories, hypotheses) and for appurtenance of concepts to other concepts (membership / indeterminacy / nonmembership). The neutrosophic triad may be *refined* by splitting $A, \mathrm{neut}A, \mathrm{anti}A$ into subentities, yielding higher-resolution appraisals without imposing $T + I + F = 1$.

Example 3.2 (Therapy Session: Revising a Work-Competence Belief (Two Updates))**.** Psychological entity:

$$e = \text{“I am competent at work.”}$$

We use the probabilistic sum $a \oplus b := a + b - ab$ and the product t-norm. Set update gains $\alpha = 0.6$ (supportive evidence), $\beta = 0.5$ (contradictory evidence), and uncertainty decay $\gamma = 0.4$ (so $1 - \gamma = 0.6$). The neutrosophic update is

$$T' = T \,\oplus\, \alpha\, e^{+}, \qquad F' = F \,\oplus\, \beta\, e^{-}, \qquad I' = \big(I \cdot (1-\gamma)\big) \,\oplus\, u.$$

Initial appraisal at $t = 0$ (before therapy homework):

$$(T_0, I_0, F_0) = (0.35,\, 0.45,\, 0.40).$$

Session inputs at $t = 0$ (positive feedback, minor criticism, some unknowns):

$$(e^{+}, e^{-}, u) = (0.70,\, 0.30,\, 0.25).$$

Step $t = 0 \to 1$ (all arithmetic shown):

$$T_1 = 0.35 \,\oplus\, (0.6 \cdot 0.70 = 0.42) = 0.35 + 0.42 - 0.35 \cdot 0.42 = 0.77 - 0.147 = 0.623,$$

$$F_1 = 0.40 \,\oplus\, (0.5 \cdot 0.30 = 0.15) = 0.40 + 0.15 - 0.40 \cdot 0.15 = 0.55 - 0.06 = 0.490,$$

$$I_1 = (0.45 \cdot 0.60 = 0.27) \,\oplus\, 0.25 = 0.27 + 0.25 - 0.27 \cdot 0.25 = 0.52 - 0.0675 = 0.4525.$$

Second-session inputs at $t = 1$ (more corroborating examples, small new uncertainty):

$$(e^{+}, e^{-}, u) = (0.60,\, 0.20,\, 0.15).$$

Step $t = 1 \to 2$:

$$T_2 = 0.623 \,\oplus\, (0.6 \cdot 0.60 = 0.36) = 0.623 + 0.36 - 0.623 \cdot 0.36 = 0.983 - 0.22428 = 0.75872,$$

$$F_2 = 0.490 \,\oplus\, (0.5 \cdot 0.20 = 0.10) = 0.490 + 0.10 - 0.490 \cdot 0.10 = 0.590 - 0.049 = 0.541,$$

$$I_2 = (0.4525 \cdot 0.60 = 0.2715) \,\oplus\, 0.15 = 0.2715 + 0.15 - 0.2715 \cdot 0.15 = 0.4215 - 0.040725 = 0.380775.$$

Interpretation. The belief's truth (expressed endorsement) rises from $0.35 \to 0.76$, uncertainty falls $0.45 \to 0.38$, while some lingering counter-belief remains ($F_2 \approx 0.54$), matching mixed but improving evidence.

Example 3.3 (Everyday Memory: "Did I Lock the Door?" (Two Updates))**.** Psychological entity:

$$e = \text{"I locked the door."}$$

Operators: $a \oplus b = a + b - ab$, product t-norm. Gains: $\alpha = 0.5$ (veridical cue), $\beta = 0.6$ (contradictory cue), $\gamma = 0.3$ (so $1 - \gamma = 0.7$).

Initial appraisal at $t = 0$ (right after leaving home):

$$(T_0, I_0, F_0) = (0.50,\ 0.30,\ 0.20).$$

Cues at $t = 0$ (mental image present, little contradiction, some uncertainty):

$$(e^+, e^-, u) = (0.60,\ 0.10,\ 0.20).$$

Step $t = 0 \to 1$:

$$\begin{aligned}
T_1 &= 0.50 \ \oplus\ (0.5 \cdot 0.60 = 0.30) = 0.50 + 0.30 - 0.50 \cdot 0.30 = 0.80 - 0.15 = 0.65,\\
F_1 &= 0.20 \ \oplus\ (0.6 \cdot 0.10 = 0.06) = 0.20 + 0.06 - 0.20 \cdot 0.06 = 0.26 - 0.012 = 0.248,\\
I_1 &= (0.30 \cdot 0.70 = 0.21) \ \oplus\ 0.20 = 0.21 + 0.20 - 0.21 \cdot 0.20 = 0.41 - 0.042 = 0.368.
\end{aligned}$$

New cues at $t = 1$ (half-remembered worry text from neighbor, attention divided; weak new positive cue):

$$(e^+, e^-, u) = (0.20,\ 0.50,\ 0.30).$$

Step $t = 1 \to 2$:

$$\begin{aligned}
T_2 &= 0.65 \ \oplus\ (0.5 \cdot 0.20 = 0.10) = 0.65 + 0.10 - 0.65 \cdot 0.10 = 0.75 - 0.065 = 0.685,\\
F_2 &= 0.248 \ \oplus\ (0.6 \cdot 0.50 = 0.30) = 0.248 + 0.30 - 0.248 \cdot 0.30 = 0.548 - 0.0744 = 0.4736,\\
I_2 &= (0.368 \cdot 0.70 = 0.2576) \ \oplus\ 0.30 = 0.2576 + 0.30 - 0.2576 \cdot 0.30 = 0.5576 - 0.07728 = 0.48032.
\end{aligned}$$

Interpretation. Competing cues push truth modestly upward ($0.50 \to 0.685$) but also increase falsity ($0.20 \to 0.474$) and uncertainty ($0.30 \to 0.480$), quantitatively capturing a realistic *memory conflict* under stress and divided attention.

3.2 Neutrosophic Memory

Neutrosophic Memory models recollection using triplets: veridical truth, indeterminacy, and falsehood, evolving under evidence, decay, cues, and retrieval thresholds, dynamically [81].

Definition 3.4 (Neutrosophic Memory)**.** [81] Let X be a nonempty universe of memory items (facts, episodes, skills) and let $\mathbb{T}$ be a time index (discrete or continuous). A *Neutrosophic Memory (NM)* on X is a family of maps

$$N(\,\cdot\,, t) : X \to [0,1]^3, \qquad x \longmapsto \big(T(x,t),\ I(x,t),\ F(x,t)\big) \quad (t \in \mathbb{T}),$$

where, for each item x and time t,

- $T(x,t)$ is the degree that x is *correctly retrievable / veridical*,
- $I(x,t)$ is the degree of *indeterminacy / uncertainty* about x,
- $F(x,t)$ is the degree that x is *falsely retrievable / distorted or non-veridical*.

The three components are independent and each lies in $[0,1]$; no normalization (such as $T+I+F = 1$) is required.

An NM is equipped with:

1. A pair $(\otimes, \oplus)$ where $\otimes : [0,1]^2 \to [0,1]$ is a t-norm and $\oplus : [0,1]^2 \to [0,1]$ is an s-norm (both commutative, associative, monotone, with $a \otimes 1 = a$, $a \oplus 0 = a$).

2. An *evolution operator* Φ_Δ such that, for $\Delta \geq 0$,

$$N(\cdot, t{+}\Delta) \;=\; \Phi_\Delta\big(N(\cdot,t),\, E^+(\cdot,t),\, E^-(\cdot,t),\, U(\cdot,t)\big),$$

where $E^+, E^-, U : X \times \mathbb{T} \to [0,1]$ encode, respectively, positive (veridical) evidence, conflicting/negative evidence, and unresolved uncertainty at time t. The family $(\Phi_\Delta)_{\Delta\geq 0}$ satisfies the semigroup laws

$$\Phi_0 = \mathrm{id}, \qquad \Phi_{\Delta_1+\Delta_2} = \Phi_{\Delta_2} \circ \Phi_{\Delta_1},$$

and is *componentwise bounded*: images remain in $[0,1]^3$.

3. A *retrieval rule* with cue similarity $s(\cdot, c) : X \to [0,1]$ (for cue c) and thresholds $(\tau, \iota, \varphi) \in [0,1]^3$:

$$\mathrm{score}_c(x,t) \;:=\; \big(T(x,t) \otimes s(x,c)\big) \;\otimes\; \big(1{-}I(x,t)\big) \;\otimes\; \big(1{-}F(x,t)\big),$$

$$\text{retrieve } x \text{ at time } t \text{ w.r.t. } c \quad\Longleftrightarrow\quad \mathrm{score}_c(x,t) \;\geq\; \tau,\;\; I(x,t) \leq \iota,\;\; F(x,t) \leq \varphi.$$

A concrete (but not mandatory) discrete-time instance is given by parameters $\alpha, \beta, \gamma, \delta_T, \delta_I, \delta_F \in [0,1]$:

$$\begin{aligned} T(x,t{+}1) &= \big((1{-}\delta_T)\, T(x,t)\big) \;\oplus\; \big(\alpha\, E^+(x,t)\big), \\ F(x,t{+}1) &= \big((1{-}\delta_F)\, F(x,t)\big) \;\oplus\; \big(\beta\, E^-(x,t)\big), \\ I(x,t{+}1) &= \big((1{-}\delta_I)\, I(x,t)\big) \;\oplus\; \big(\gamma\, U(x,t)\big), \end{aligned}$$

which preserves $[0,1]$ and allows T, I, F to evolve independently.

Example 3.5 (Password Recall under Stress (Two Updates + Retrieval))**.** Memory item:

$$x = \text{“My email password } \texttt{p@ss123} \text{ is correct.”}$$

Operators: probabilistic sum $a \oplus b := a + b - ab$; product t-norm. Parameters: $\alpha = 0.6$ (veridical cue gain), $\beta = 0.5$ (contradictory cue gain), $\gamma = 0.4$, and decays $(\delta_T, \delta_F, \delta_I) = (0.1, 0.1, 0.2)$.

Update equations:

$$T' = (1{-}\delta_T)T \;\oplus\; \alpha\, E^+, \qquad F' = (1{-}\delta_F)F \;\oplus\; \beta\, E^-, \qquad I' = (1{-}\delta_I)I \;\oplus\; \gamma\, U.$$

Initial appraisal at $t = 0$:

$$(T_0, I_0, F_0) = (0.40,\, 0.35,\, 0.30).$$

Cues at $t = 0$: $(E^+, E^-, U) = (0.60,\, 0.20,\, 0.25)$.

Step $t = 0 \to 1$ (explicit arithmetic):

$$\begin{aligned} T_1 &= (0.9 \cdot 0.40) \;\oplus\; (0.6 \cdot 0.60) = 0.36 \;\oplus\; 0.36 = 0.36 + 0.36 - 0.36 \cdot 0.36 = 0.5904, \\ F_1 &= (0.9 \cdot 0.30) \;\oplus\; (0.5 \cdot 0.20) = 0.27 \;\oplus\; 0.10 = 0.27 + 0.10 - 0.27 \cdot 0.10 = 0.3430, \\ I_1 &= (0.8 \cdot 0.35) \;\oplus\; (0.4 \cdot 0.25) = 0.28 \;\oplus\; 0.10 = 0.28 + 0.10 - 0.28 \cdot 0.10 = 0.3520. \end{aligned}$$

Cues at $t = 1$: $(E^+, E^-, U) = (0.70,\, 0.40,\, 0.15)$.

Step $t = 1 \to 2$:

$$\begin{aligned} T_2 &= (0.9 \cdot 0.5904) \;\oplus\; (0.6 \cdot 0.70) = 0.53136 \;\oplus\; 0.42 \\ &= 0.53136 + 0.42 - 0.53136 \cdot 0.42 = 0.95136 - 0.2231712 = 0.7281888, \\ F_2 &= (0.9 \cdot 0.3430) \;\oplus\; (0.5 \cdot 0.40) = 0.3087 \;\oplus\; 0.20 = 0.5087 - 0.06174 = 0.44696, \\ I_2 &= (0.8 \cdot 0.3520) \;\oplus\; (0.4 \cdot 0.15) = 0.2816 \;\oplus\; 0.06 = 0.3416 - 0.016896 = 0.324704. \end{aligned}$$

Retrieval with cue similarity $s(x, c) = 0.85$ (browser/keyboard context) and product t-norm:

$$\mathrm{score}_c(x, 2) = (T_2 s) \cdot (1{-}I_2) \cdot (1{-}F_2).$$

Numerics:

$$\begin{aligned} T_2 s &= 0.7281888 \cdot 0.85 = 0.61896048, \\ 1{-}I_2 &= 1 - 0.324704 = 0.675296, \qquad 1{-}F_2 = 1 - 0.44696 = 0.55304, \\ \mathrm{score}_c &= 0.61896048 \cdot 0.675296 \cdot 0.55304 \approx 0.4179815363 \cdot 0.55304 \approx 0.2311605. \end{aligned}$$

Decision (thresholds $\tau = 0.25$, $\iota = 0.50$, $\varphi = 0.60$): $I_2 = 0.325 \leq 0.50$, $F_2 = 0.447 \leq 0.60$, but score$= 0.231 < 0.25$; retrieval *fails*, matching the user's hesitancy despite partial evidence.

Example 3.6 (Where Did I Park the Car? (Two Updates + Retrieval))**.** Memory item:

$$x = \text{“Parked on Level B2, Zone Green.”}$$

Operators: $a \oplus b = a + b - ab$, product t-norm. Parameters: $\alpha = 0.7, \beta = 0.6, \gamma = 0.5$, and $(\delta_T, \delta_F, \delta_I) = (0.1, 0.1, 0.25)$.

Initial appraisal at $t = 0$:

$$(T_0, I_0, F_0) = (0.30,\ 0.50,\ 0.20).$$

Cues at $t = 0$ (photo of pillar B2-G, friend suggests B3, some uncertainty):

$$(E^+, E^-, U) = (0.80,\ 0.20,\ 0.30).$$

Step $t = 0 \to 1$:

$$\begin{aligned} T_1 &= (0.9 \cdot 0.30) \ \oplus\ (0.7 \cdot 0.80) = 0.27 \ \oplus\ 0.56 = 0.83 - 0.1512 = 0.6788, \\ F_1 &= (0.9 \cdot 0.20) \ \oplus\ (0.6 \cdot 0.20) = 0.18 \ \oplus\ 0.12 = 0.30 - 0.0216 = 0.2784, \\ I_1 &= (0.75 \cdot 0.50) \ \oplus\ (0.5 \cdot 0.30) = 0.375 \ \oplus\ 0.15 = 0.525 - 0.05625 = 0.46875. \end{aligned}$$

Cues at $t = 1$ (see signs/landmarks; mild confusion; less uncertainty):

$$(E^+, E^-, U) = (0.50,\ 0.10,\ 0.10).$$

Step $t = 1 \to 2$:

$$\begin{aligned} T_2 &= (0.9 \cdot 0.6788) \ \oplus\ (0.7 \cdot 0.50) = 0.61092 \ \oplus\ 0.35 = 0.96092 - 0.213822 = 0.747098, \\ F_2 &= (0.9 \cdot 0.2784) \ \oplus\ (0.6 \cdot 0.10) = 0.25056 \ \oplus\ 0.06 = 0.31056 - 0.0150336 = 0.2955264, \\ I_2 &= (0.75 \cdot 0.46875) \ \oplus\ (0.5 \cdot 0.10) = 0.3515625 \ \oplus\ 0.05 = 0.4015625 - 0.017578125 = 0.383984375. \end{aligned}$$

Retrieval with cue similarity $s(x, c) = 0.90$ (matching color/level cues):

$$\begin{aligned} T_2 s &= 0.747098 \cdot 0.90 = 0.6723882, \\ 1{-}I_2 &= 1 - 0.383984375 = 0.616015625, \qquad 1{-}F_2 = 1 - 0.2955264 = 0.7044736, \\ \text{score}_c &= 0.6723882 \cdot 0.616015625 \cdot 0.7044736 \approx 0.4142016373 \cdot 0.7044736 \approx 0.2917941. \end{aligned}$$

Decision (thresholds $\tau = 0.28$, $\iota = 0.50$, $\varphi = 0.50$): $I_2 = 0.384 \le 0.50$, $F_2 = 0.296 \le 0.50$, and score$= 0.292 \ge 0.28$; retrieval *succeeds*, consistent with finding the car using corroborating landmarks.

3.3 Neutrosophic Body-Soul-Mind Functioning

Neutrosophic Body-Soul-Mind Functioning models personal functioning across body, soul, mind using triplet truth, indeterminacy, falsehood with couplings, updates, over time [81, 88].

Definition 3.7 (Neutrosophic Body-Soul-Mind Functioning (NBSMF))**.** [81] Let P be a nonempty set of persons, $\mathbb{T}$ a time index (discrete or continuous), and

$$C := \{B, S, M\} \quad \text{(Body, Soul, Mind)}.$$

For each person $p \in P$, facet $c \in C$, and time $t \in \mathbb{T}$, a *neutrosophic functioning state* is a triplet

$$N_c(p,t) \ := \ \big(T_c(p,t),\ I_c(p,t),\ F_c(p,t)\big) \ \in \ [0,1]^3,$$

where (i) T_c is the degree that *p functions well* on facet c at time t, (ii) I_c is the degree of *indeterminacy/uncertainty* about that functioning, (iii) F_c is the degree that *p fails* on facet c. The components are independent and no normalization (e.g. $T_c + I_c + F_c = 1$) is required.

Facet Coupling and Evolution. Let $A = (a_{d\to c})_{c,d\in C}$ be a coupling matrix with $a_{d\to c} \in [0, 1]$ ($a_{c\to c}$ allowed) encoding how facet d influences facet c. Let $(\otimes, \oplus)$ be a fixed t-norm and s-norm on $[0, 1]$ (commutative, associative, monotone,

with $u \otimes 1 = u$ and $u \oplus 0 = u$). Given supportive/impairing/uncertainty inputs $e_c^+, e_c^-, u_c : P \times \mathbb{T} \to [0,1]$ and parameters $\alpha_c, \beta_c, \gamma_c, \delta_c^T, \delta_c^F, \delta_c^I \in [0,1]$, define one lawful discrete-time update by

$$T_c(p,t{+}1) = \big((1{-}\delta_c^T)\, T_c(p,t)\big) \;\oplus\; \big(\alpha_c\, e_c^+(p,t)\big) \;\oplus\; \underbrace{\Big(1 - \prod_{d \in C} \big(1 - a_{d\to c}\, T_d(p,t)\big)\Big)}_{\text{synergy from other facets}},$$

$$F_c(p,t{+}1) = \big((1{-}\delta_c^F)\, F_c(p,t)\big) \;\oplus\; \big(\beta_c\, e_c^-(p,t)\big) \;\oplus\; \underbrace{\prod_{d \in C} \big(F_d(p,t)\big)^{a_{d\to c}}}_{\text{propagated failure}},$$

$$I_c(p,t{+}1) = \big((1{-}\delta_c^I)\, I_c(p,t)\big) \;\oplus\; \big(\gamma_c\, u_c(p,t)\big).$$

This keeps all components in $[0,1]$ and permits T_c, I_c, F_c to evolve independently.

Person-Level Aggregation. Let $w_c(p) \in [0,1]$ be facet salience weights with $\sum_c w_c(p) \le 3$ (no normalization required). Define the overall functioning of p at time t by

$$\begin{aligned}
T_*(p,t) &:= 1 - \prod_{c \in C} \big(1 - w_c(p)\, T_c(p,t)\big), \\
I_*(p,t) &:= \mathsf{A}_I\big(\{\, w_c(p)\, I_c(p,t) : c \in C \,\}\big), \qquad \mathsf{A}_I : \text{ any monotone aggregator (e.g. weighted mean or } \max). \\
F_*(p,t) &:= \prod_{c \in C} \big(F_c(p,t)\big)^{w_c(p)},
\end{aligned}$$

Task-/Goal-Level Evaluation. Let $\mathcal{G}$ be a set of goals, and $\kappa_c : \mathcal{G} \to [0,1]$ encode each goal's reliance on facet c. The goal-specific functioning of p for $g \in \mathcal{G}$ is

$$\begin{aligned}
T_g(p,t) &:= 1 - \prod_{c \in C} \big(1 - \kappa_c(g)\, w_c(p)\, T_c(p,t)\big), \\
I_g(p,t) &:= \mathsf{A}_I\big(\{\, \kappa_c(g)\, w_c(p)\, I_c(p,t) : c \in C \,\}\big), \\
F_g(p,t) &:= \prod_{c \in C} \big(F_c(p,t)\big)^{\kappa_c(g)\, w_c(p)}.
\end{aligned}$$

Given thresholds $(\tau, \iota, \varphi) \in [0,1]^3$, we set

$$p \models_{(\tau,\iota,\varphi)} g \quad \Longleftrightarrow \quad T_g(p,t) \ge \tau,\;\; I_g(p,t) \le \iota,\;\; F_g(p,t) \le \varphi.$$

Basic Properties (checked by inequalities). Assume the probabilistic sum $a \oplus b := a + b - ab$. For any p, t and any facet/goal:

$$0 \le T_*, I_*, F_* \le 1, \qquad 0 \le T_g, I_g, F_g \le 1.$$

Indeed, for T_*: each factor $1 - w_c T_c \in [0,1]$, hence $\prod_c (1 - w_c T_c) \in [0,1]$ and $T_* = 1 - \prod_c (1 - w_c T_c) \in [0,1]$. For F_*: each $(F_c)^{w_c} \in [0,1]$, so their product is in $[0,1]$. For I_*: monotonicity of A_I keeps values in $[0,1]$. For the updates, write $a = (1{-}\delta)\, X \in [0,1]$ and $b = \eta\, Z \in [0,1]$; then $a \oplus b = a + b - ab \in [0,1]$, and $\partial(a \oplus b)/\partial b = 1 - a \ge 0$, so $T_c(t{+}1)$ is nondecreasing in e_c^+ and the synergy term; similarly $F_c(t{+}1)$ is nondecreasing in e_c^- and the propagated-failure term.

Example 3.8 (Workday Readiness: Interplay of Body-Soul-Mind (Two Updates))**. Person:** Hana. Facets $C = \{B, S, M\}$ (Body, Soul, Mind). Use $\oplus$ the probabilistic sum $a \oplus b = a + b - ab$, and product t-norm. Common parameters: $\alpha = 0.6$, $\beta = 0.5$, $\gamma = 0.4$, $(\delta^T, \delta^F, \delta^I) = (0.1, 0.1, 0.2)$. Couplings $A = (a_{d\to c})$ (nonzeros; zeros on the diagonal):

$$a_{M\to B} = 0.2,\; a_{S\to B} = 0.1; \quad a_{B\to S} = 0.2,\; a_{M\to S} = 0.3; \quad a_{B\to M} = 0.3,\; a_{S\to M} = 0.2.$$

Initial state at $t = 0$:

c	$T_c(0)$	$I_c(0)$	$F_c(0)$
B	0.60	0.20	0.20
S	0.50	0.30	0.20
M	0.40	0.40	0.30

Inputs at $t = 0$:

$$(e_B^+, e_B^-, u_B) = (0.7, 0.2, 0.1),\; (e_S^+, e_S^-, u_S) = (0.6, 0.1, 0.2),\; (e_M^+, e_M^-, u_M) = (0.5, 0.3, 0.2).$$

Synergy and failure propagation terms at $t = 0$:

$$\begin{aligned}
\mathrm{syn}_B &= 1 - (1 - 0.1T_S)(1 - 0.2T_M) = 1 - (1 - 0.05)(1 - 0.08) = 0.126,\\
\mathrm{syn}_S &= 1 - (1 - 0.2T_B)(1 - 0.3T_M) = 1 - (1 - 0.12)(1 - 0.12) = 0.2256,\\
\mathrm{syn}_M &= 1 - (1 - 0.3T_B)(1 - 0.2T_S) = 1 - (1 - 0.18)(1 - 0.10) = 0.262,\\
\mathrm{propF}_B &= (F_M)^{0.2}(F_S)^{0.1} = 0.3^{0.2}\,0.2^{0.1} = 0.66916,\\
\mathrm{propF}_S &= (F_B)^{0.2}(F_M)^{0.3} = 0.2^{0.2}\,0.3^{0.3} = 0.50506,\\
\mathrm{propF}_M &= (F_B)^{0.3}(F_S)^{0.2} = 0.2^{0.3}\,0.2^{0.2} = 0.44721.
\end{aligned}$$

Update $t = 0 \to 1$ (componentwise):

$$\begin{aligned}
T_B(1) &= (0.9 \cdot 0.60) \oplus (0.6 \cdot 0.70) \oplus \mathrm{syn}_B = 0.54 \oplus 0.42 \oplus 0.126 \Rightarrow T_B(1) = 0.7668168,\\
F_B(1) &= (0.9 \cdot 0.20) \oplus (0.5 \cdot 0.20) \oplus \mathrm{propF}_B = 0.18 \oplus 0.10 \oplus 0.66916 \Rightarrow F_B(1) = 0.7558369848,\\
I_B(1) &= (0.8 \cdot 0.20) \oplus (0.4 \cdot 0.10) = 0.16 \oplus 0.04 \Rightarrow I_B(1) = 0.1936;
\end{aligned}$$

(similarly)

$$\begin{aligned}
&T_S(1) = 0.7274112, \quad F_S(1) = 0.6144411975, \quad I_S(1) = 0.3008,\\
&T_M(1) = 0.6693760, \quad F_M(1) = 0.6569960360, \quad I_M(1) = 0.3744.
\end{aligned}$$

Inputs at $t = 1$:

$$(e_B^+, e_B^-, u_B) = (0.5, 0.3, 0.1),\ (e_S^+, e_S^-, u_S) = (0.5, 0.2, 0.1),\ (e_M^+, e_M^-, u_M) = (0.6, 0.2, 0.1).$$

Update $t = 1 \to 2$ (final values):

c	$T_c(2)$	$I_c(2)$	$F_c(2)$
B	0.8257985076	0.1886848	0.9662200287
S	0.8364397269	0.2710144	0.9330543301
M	0.8325940776	0.3275392	0.9389809817

Person-level aggregation at $t = 2$ (weights w_B=0.4, w_S=0.3, w_M=0.3):

$$\begin{aligned}
T_* &= 1 - \prod_{c \in \{B,S,M\}} (1 - w_c T_c) = 0.6236614785,\\
I_* &= \sum_c w_c I_c = 0.2550400, \qquad F_* = \prod_c (F_c)^{w_c} = 0.9479807814.
\end{aligned}$$

Interpretation: Strong cross-facet reinforcement lifts T on all three facets; at the same time, contra-signals propagate as high F (neutrosophically allowed: T and F are independent).

Example 3.9 (Half-Marathon Training Day (Two Updates))**. Person:** Ken. Same operators as Example 1. Parameters: $\alpha = 0.5, \beta = 0.3, \gamma = 0.3$, $(\delta^T, \delta^F, \delta^I) = (0.1, 0.1, 0.2)$. Couplings (nonzeros; zeros on the diagonal):

$$a_{M\to B} = 0.10,\ a_{S\to B} = 0.20; \quad a_{B\to S} = 0.10,\ a_{M\to S} = 0.10; \quad a_{B\to M} = 0.25,\ a_{S\to M} = 0.25.$$

Initial state at $t = 0$:

c	$T_c(0)$	$I_c(0)$	$F_c(0)$
B	0.50	0.30	0.10
S	0.60	0.25	0.05
M	0.50	0.35	0.10

Inputs at $t = 0$:

$$(e_B^+, e_B^-, u_B) = (0.7, 0.05, 0.1),\ (e_S^+, e_S^-, u_S) = (0.6, 0.05, 0.1),\ (e_M^+, e_M^-, u_M) = (0.5, 0.05, 0.2).$$

Synergy / failure-propagation at $t = 0$ (numerics):

$$\mathrm{syn}_B = 0.1640,\ \mathrm{syn}_S = 0.0975,\ \mathrm{syn}_M = 0.25625; \quad \mathrm{propF}_B = 0.43631,\ \mathrm{propF}_S = 0.63096,\ \mathrm{propF}_M = 0.26591.$$

Update $t = 0 \to 1$ (final values):

c	$T_c(1)$	$I_c(1)$	$F_c(1)$
B	0.7011300	0.2628000	0.4947354185
S	0.7093950	0.2240000	0.6528508000
M	0.6932031250	0.3232000	0.3420027264

Inputs at $t = 1$:

$$(e_B^+, e_B^-, u_B) = (0.6, 0.05, 0.05),\ (e_S^+, e_S^-, u_S) = (0.5, 0.05, 0.05),\ (e_M^+, e_M^-, u_M) = (0.6, 0.05, 0.10).$$

Update $t = 1 \to 2$ (final values):

c	$T_c(2)$	$I_c(2)$	$F_c(2)$
B	0.7937218807	0.2220864	0.9042854707
S	0.7653322366	0.1915120	0.9338715076
M	0.8213750205	0.2808032	0.8321853019

Person-level aggregation at $t = 2$ (weights w_B=0.5, w_S=0.2, w_M=0.3):

$$T_* = 1 - \prod_c (1 - w_c T_c) = 0.6150534111,$$
$$I_* = \sum_c w_c I_c = 0.23358656, \qquad F_* = \prod_c (F_c)^{w_c} = 0.8877204280.$$

Interpretation: Training and social support raise T across facets; propagated risks (fatigue, doubt) keep F sizable. The model cleanly tracks simultaneous reinforcement and lingering failure/uncertainty signals without forcing $T+I+F = 1$.

3.4 Neutrosophic Psychological System

Neutrosophic Psychological System models interacting psychological variables via triplet truth, indeterminacy, falsehood; updates under evidence, coupling, uncertainty, generating behaviors dynamically [81].

Definition 3.10 (Neutrosophic Psychological System (NPS))**.** [81] Fix a nonempty, finite set of psychological variables E (e.g., beliefs, emotions, drives, memory traces), a time index $\mathbb{T}$ (discrete or continuous), and write $[0,1]^3 \ni (T, I, F)$ for (neutrosophic) truth/membership, indeterminacy, and falsehood/nonmembership, respectively.

An *NPS* is a tuple

$$\mathfrak{N} = \big(E,\ \mathbb{T},\ \mathcal{S},\ \Phi,\ A,\ \oplus,\ \otimes,\ \mathsf{A}_I,\ B,\ \kappa\big),$$

consisting of:

- the *state space* $\mathcal{S} := ([0,1]^3)^E$, whose elements are maps $N(\cdot, t) : E \to [0,1]^3$, $e \mapsto \big(T_e(t), I_e(t), F_e(t)\big)$;
- a family of *evolution operators* $(\Phi_\Delta)_{\Delta \in \mathbb{T}_+}$ with
$$N(\cdot, t{+}\Delta) = \Phi_\Delta\big(N(\cdot, t), U(t)\big), \qquad \Phi_0 = \mathrm{id},\ \ \Phi_{\Delta_1+\Delta_2} = \Phi_{\Delta_2} \circ \Phi_{\Delta_1},$$
where $U(t)$ encodes exogenous inputs (stimuli/evidence/uncertainty) at time t;
- a (possibly weighted) *coupling matrix* $A = (a_{f\to e})_{e,f\in E}$ with $a_{f\to e} \in [0,1]$ describing how f influences e;
- a fixed t-norm $\otimes$ and s-norm $\oplus$ on $[0,1]$ (commutative, associative, monotone, with $u \otimes 1 = u$ and $u \oplus 0 = u$), and a monotone *indeterminacy aggregator* A_I on $[0,1]$ (e.g., weighted mean or max);
- a (finite) set of *behaviors/outputs* B with reliance weights $\kappa_e(b) \in [0,1]$ for $b \in B$, $e \in E$.

Componentwise lawful update (one concrete instance). Let $U(t) = (e_e^+(t), e_e^-(t), u_e(t))_{e\in E} \in [0,1]^{3|E|}$ be, respectively, supportive (pro) evidence, contradictory (contra) evidence, and unresolved uncertainty at time t, and let $\alpha_e, \beta_e, \gamma_e, \delta_e^T, \delta_e^F, \delta_e^I \in [0,1]$ be parameters. A valid discrete-time Φ_1 is given componentwise by

$$T_e(t{+}1) = \big((1{-}\delta_e^T)\, T_e(t)\big) \ \oplus\ \big(\alpha_e\, e_e^+(t)\big) \ \oplus\ \Big(1 - \prod_{f\in E}\big(1 - a_{f\to e}\, T_f(t)\big)\Big),$$
$$F_e(t{+}1) = \big((1{-}\delta_e^F)\, F_e(t)\big) \ \oplus\ \big(\beta_e\, e_e^-(t)\big) \ \oplus\ \prod_{f\in E}\big(F_f(t)\big)^{a_{f\to e}},$$
$$I_e(t{+}1) = \big((1{-}\delta_e^I)\, I_e(t)\big) \ \oplus\ \big(\gamma_e\, u_e(t)\big),$$

which preserves $[0,1]$ and does not require $T_e+I_e+F_e = 1$.

Output/behavior map. For each $b \in B$, define neutrosophic output degrees by

$$T_b(t) := 1 - \prod_{e\in E}\big(1 - \kappa_e(b)\, T_e(t)\big),$$
$$I_b(t) := \mathsf{A}_I\big(\{\, \kappa_e(b)\, I_e(t) : e \in E \,\}\big),$$
$$F_b(t) := \prod_{e\in E}\big(F_e(t)\big)^{\kappa_e(b)}.$$

A (thresholded) decision rule at $(\tau,\iota,\varphi) \in [0,1]^3$ declares

$$\text{“}b \text{ is expressed at time } t\text{”} \quad\Longleftrightarrow\quad T_b(t) \ge \tau,\ \ I_b(t) \le \iota,\ \ F_b(t) \le \varphi.$$

Axioms. (i) *Semigroup/causality*: $(\Phi_\Delta)_{\Delta\ge 0}$ satisfies the semigroup law above; (ii) *Boundedness*: $\Phi_\Delta(\mathcal{S}) \subseteq \mathcal{S}$; (iii) *Isomorphism invariance*: relabelings of E preserving A yield conjugate trajectories.

Example 3.11 (Job Interview (Preparedness-Calmness-Recall) as an NPS, Two Updates)**.** $E = \{\mathrm{P},\mathrm{C},\mathrm{R}\}$ with P = preparedness, C = calmness, R = recall ability. Use the probabilistic sum $a \oplus b := a + b - ab$ and the product t-norm.

Couplings. (nonzeros; others 0)

$$a_{\mathrm{P}\to\mathrm{R}} = 0.4,\quad a_{\mathrm{C}\to\mathrm{R}} = 0.3,\quad a_{\mathrm{P}\to\mathrm{C}} = 0.2,\quad a_{\mathrm{C}\to\mathrm{P}} = 0.2,\quad a_{\mathrm{R}\to\mathrm{P}} = 0.1.$$

Parameters. For each $e \in E$:

$$\alpha_e = 0.6,\ \beta_e = 0.5,\ \gamma_e = 0.4,\quad \delta_e^T = 0.1,\ \delta_e^F = 0.1,\ \delta_e^I = 0.2.$$

Initial state (t=0).

e	$T_e(0)$	$I_e(0)$	$F_e(0)$
P	0.50	0.30	0.30
C	0.40	0.35	0.40
R	0.45	0.40	0.35

Inputs at t=0.

$$(e_{\mathrm{P}}^+, e_{\mathrm{P}}^-, u_{\mathrm{P}}) = (0.7, 0.2, 0.2),\quad (e_{\mathrm{C}}^+, e_{\mathrm{C}}^-, u_{\mathrm{C}}) = (0.5, 0.3, 0.2),\quad (e_{\mathrm{R}}^+, e_{\mathrm{R}}^-, u_{\mathrm{R}}) = (0.6, 0.2, 0.2).$$

Synergy/propagated-failure at t=0. For $e \in E$,

$$\mathrm{syn}_e = 1 - \prod_{f\in E}\big(1 - a_{f\to e} T_f(0)\big),$$
$$\mathrm{propF}_e = \prod_{f\in E}\big(F_f(0)\big)^{a_{f\to e}}.$$

Numerically:

$$\mathrm{syn}_{\mathrm{P}} = 0.121400,\ \mathrm{syn}_{\mathrm{C}} = 0.100000,\ \mathrm{syn}_{\mathrm{R}} = 0.296000;\quad \mathrm{propF}_{\mathrm{P}} = 0.749581,\ \mathrm{propF}_{\mathrm{C}} = 0.786003,\ \mathrm{propF}_{\mathrm{R}} = 0.469317.$$

Update $t \to t$+1. The NPS law (componentwise) is

$$T_e' = \big((1-\delta_e^T)T_e\big) \ \oplus\ \alpha_e e_e^+ \ \oplus\ \mathrm{syn}_e,$$
$$F_e' = \big((1-\delta_e^F)F_e\big) \ \oplus\ \beta_e e_e^- \ \oplus\ \mathrm{propF}_e,$$
$$I_e' = \big((1-\delta_e^I)I_e\big) \ \oplus\ \gamma_e u_e.$$

For P (all arithmetic shown):

$$\begin{aligned} T_{\mathrm{P}}(1) &= (0.9\cdot 0.50) \ \oplus\ (0.6\cdot 0.70) \ \oplus\ 0.121400 \\ &= 0.45 \ \oplus\ 0.42 \ \oplus\ 0.121400 = 0.7197266, \\ F_{\mathrm{P}}(1) &= (0.9\cdot 0.30) \ \oplus\ (0.5\cdot 0.20) \ \oplus\ 0.749581 \\ &= 0.27 \ \oplus\ 0.10 \ \oplus\ 0.749581 = 0.8354749812, \\ I_{\mathrm{P}}(1) &= (0.8\cdot 0.30) \ \oplus\ (0.4\cdot 0.20) = 0.24 \ \oplus\ 0.08 = 0.3008. \end{aligned}$$

Analogously,

$$T_{\mathrm{C}}(1) = 0.596800, \quad I_{\mathrm{C}}(1) = 0.337600, \quad F_{\mathrm{C}}(1) = 0.8835856786,$$
$$T_{\mathrm{R}}(1) = 0.7319168000, \quad I_{\mathrm{R}}(1) = 0.374400, \quad F_{\mathrm{R}}(1) = 0.6728340727.$$

Inputs at t=1.

$$(e_{\mathrm{P}}^{+}, e_{\mathrm{P}}^{-}, u_{\mathrm{P}}) = (0.6, 0.1, 0.1),\ (e_{\mathrm{C}}^{+}, e_{\mathrm{C}}^{-}, u_{\mathrm{C}}) = (0.7, 0.2, 0.15),\ (e_{\mathrm{R}}^{+}, e_{\mathrm{R}}^{-}, u_{\mathrm{R}}) = (0.5, 0.1, 0.1).$$

Update t=1 → 2 (final state).

e	$T_e(2)$	$I_e(2)$	$F_e(2)$
P	0.816001	0.271014	0.985306
C	0.770175	0.313875	0.993492
R	0.860340	0.327539	0.961291

Behavior. b = "Answer technical questions clearly." Reliance weights $\kappa_{\mathrm{P}} = 0.40,\ \kappa_{\mathrm{C}} = 0.25,\ \kappa_{\mathrm{R}} = 0.35.$

$$T_b = 1 - \prod_{e \in E} \big(1 - \kappa_e T_e(2)\big) = 1 - (0.673599)(0.807456)(0.698881)$$
$$= 0.619877,$$
$$I_b = \sum_{e} \kappa_e I_e(2) = 0.40 \cdot 0.271014 + 0.25 \cdot 0.313875 + 0.35 \cdot 0.327539 = 0.301513,$$
$$F_b = \prod_{e} \big(F_e(2)\big)^{\kappa_e} = \exp\Big(\sum_{e} \kappa_e \ln F_e(2)\Big) = 0.978856.$$

Decision at thresholds $(\tau, \iota, \varphi) = (0.60, 0.35, 0.98)$*:* $T_b = 0.620 \geq 0.60,\ I_b = 0.302 \leq 0.35,\ F_b = 0.979 \leq 0.98 \Rightarrow$ *expressed.*

Example 3.12 (Daily Smoking Abstinence as an NPS (Morning→Afternoon))**.** $E = \{\mathrm{Mot}, \mathrm{Ctrl}, \mathrm{Supp}\}$ with motivation, craving control, social support. Same operators as above.

Couplings. (nonzeros)

$$a_{\mathrm{Supp}\to\mathrm{Mot}} = 0.2, \quad a_{\mathrm{Mot}\to\mathrm{Ctrl}} = 0.15, \quad a_{\mathrm{Supp}\to\mathrm{Ctrl}} = 0.1, \quad a_{\mathrm{Mot}\to\mathrm{Supp}} = 0.05, \quad a_{\mathrm{Ctrl}\to\mathrm{Supp}} = 0.05.$$

Parameters.

$$\alpha_{\mathrm{Mot}}{=}0.55,\ \alpha_{\mathrm{Ctrl}}{=}0.50,\ \alpha_{\mathrm{Supp}}{=}0.45, \quad \beta_{\mathrm{Mot}}{=}0.35,\ \beta_{\mathrm{Ctrl}}{=}0.40,\ \beta_{\mathrm{Supp}}{=}0.30,$$
$$\gamma_e = 0.30\ (e \in E), \qquad \delta_e^{T} = 0.1,\ \delta_e^{F} = 0.1,\ \delta_e^{I} = 0.2.$$

Initial state (morning t=0).

e	$T_e(0)$	$I_e(0)$	$F_e(0)$
Mot	0.60	0.35	0.20
Ctrl	0.50	0.30	0.25
Supp	0.55	0.25	0.20

Morning inputs.

$$(e_{\mathrm{Mot}}^{+}, e_{\mathrm{Mot}}^{-}, u_{\mathrm{Mot}}) = (0.7, 0.1, 0.2),$$
$$(e_{\mathrm{Ctrl}}^{+}, e_{\mathrm{Ctrl}}^{-}, u_{\mathrm{Ctrl}}) = (0.5, 0.3, 0.2),$$
$$(e_{\mathrm{Supp}}^{+}, e_{\mathrm{Supp}}^{-}, u_{\mathrm{Supp}}) = (0.6, 0.1, 0.15).$$

Synergy/propagated-failure at t=0 (numerics).

$$\mathrm{syn}_{\mathrm{Mot}} = 0.110000, \quad \mathrm{syn}_{\mathrm{Ctrl}} = 0.140050, \quad \mathrm{syn}_{\mathrm{Supp}} = 0.054250;$$
$$\mathrm{propF}_{\mathrm{Mot}} = 0.724780,\ \mathrm{propF}_{\mathrm{Ctrl}} = 0.668740,\ \mathrm{propF}_{\mathrm{Supp}} = 0.860892.$$

Update to late morning t=1.

e	$T_e(1)$	$I_e(1)$	$F_e(1)$
Mot	0.748219	0.323200	0.782218
Ctrl	0.645271	0.285600	0.774081
Supp	0.651349	0.236000	0.889353

Afternoon inputs (t=1).

$$(e^+_{\text{Mot}}, e^-_{\text{Mot}}, u_{\text{Mot}}) = (0.6, 0.1, 0.1),$$
$$(e^+_{\text{Ctrl}}, e^-_{\text{Ctrl}}, u_{\text{Ctrl}}) = (0.6, 0.2, 0.1),$$
$$(e^+_{\text{Supp}}, e^-_{\text{Supp}}, u_{\text{Supp}}) = (0.5, 0.1, 0.1).$$

Update to afternoon t=2 (final state).

e	$T_e(2)$	$I_e(2)$	$F_e(2)$
Mot	0.809682	0.280803	0.993379
Ctrl	0.756429	0.251626	0.986770
Supp	0.701273	0.213136	0.995204

Behavior. b = "Abstain from smoking today." Reliance weights $\kappa_{\text{Mot}} = 0.50,\ \kappa_{\text{Ctrl}} = 0.30,\ \kappa_{\text{Supp}} = 0.20.$

$$\begin{aligned} T_b &= 1 - \prod_{e \in E} \big(1 - \kappa_e T_e(2)\big) = 1 - (1 - 0.50 \cdot 0.809682)(1 - 0.30 \cdot 0.756429)(1 - 0.20 \cdot 0.701273) \\ &= 1 - (0.595159)(0.773071)(0.859745) = 0.604431, \\ I_b &= \sum_e \kappa_e I_e(2) = 0.50 \cdot 0.280803 + 0.30 \cdot 0.251626 + 0.20 \cdot 0.213136 = 0.258516, \\ F_b &= \prod_e \big(F_e(2)\big)^{\kappa_e} = \exp\Big(0.50 \ln 0.993379 + 0.30 \ln 0.986770 + 0.20 \ln 0.995204\Big) = 0.991756. \end{aligned}$$

Decision at thresholds $(\tau, \iota, \varphi) = (0.60, 0.35, 0.995)$*:* $T_b = 0.604 \geq 0.60,\ I_b = 0.259 \leq 0.35,\ F_b = 0.992 \leq 0.995 \Rightarrow$ *expressed.*

3.5 Neutrosophic Personality

Neutrosophic Personality models trait expression using the triplet of truth, indeterminacy, and falsehood, aggregating indicators across contexts and time with update rules [89, 90]. Related studies also include Personality models [91–93] and Fuzzy Personality models [94–96].

Definition 3.13 (Neutrosophic Personality (NP))**.** [89, 90] Let P be a nonempty set of persons, D a nonempty set of personality dimensions (traits), C a set of contexts (situations), and $\mathbb{T}$ a time index (discrete or continuous). For each trait $d \in D$, context $c \in C$, and time $t \in \mathbb{T}$, a *neutrosophic personality state* is the map

$$N_d(\,\cdot\,, c, t) : P \longrightarrow [0,1]^3, \qquad p \longmapsto \big(T_d(p,c,t),\ I_d(p,c,t),\ F_d(p,c,t)\big),$$

where, for each person $p \in P$,

- $T_d(p,c,t)$ is the degree that p *expresses* trait d in context c at time t,
- $I_d(p,c,t)$ is the degree of *indeterminacy/uncertainty* about that expression,
- $F_d(p,c,t)$ is the degree that p *does not express* (or expresses the opposite of) trait d.

The three components are independent and each lies in $[0,1]$; no normalization such as $T_d{+}I_d{+}F_d = 1$ is required.

Measurement model and trait aggregation. Fix for each $d \in D$ a finite set of indicators/items Q_d (e.g., questionnaire items or behavioral cues) with nonnegative weights $w_{d,q} \in [0,1]$. Each item $q \in Q_d$ admits a neutrosophic appraisal for p in context c at time t:

$$n_q(p,c,t) \ = \ \big(T_q(p,c,t),\ I_q(p,c,t),\ F_q(p,c,t)\big) \in [0,1]^3.$$

Let A_I be any monotone aggregator on $[0,1]$ (e.g., weighted mean or max). The trait-level neutrosophic degrees are defined by

$$\begin{aligned} T_d(p,c,t) &:= 1 - \prod_{q \in Q_d} \big(1 - w_{d,q}\, T_q(p,c,t)\big), \\ I_d(p,c,t) &:= \mathsf{A}_I\big(\{\, w_{d,q}\, I_q(p,c,t) : q \in Q_d \,\}\big), \\ F_d(p,c,t) &:= \prod_{q \in Q_d} \big(F_q(p,c,t)\big)^{w_{d,q}}. \end{aligned}$$

The *neutrosophic personality profile* of p in context c at time t is the vector

$$\mathrm{NP}(p,c,t) \;:=\; \big(N_d(p,c,t)\big)_{d\in D} \;\in\; \big([0,1]^3\big)^D.$$

Decision (expression) rule. Given thresholds $(\tau,\iota,\varphi)\in[0,1]^3$, we say that p *expresses* trait d at (c,t) when

$$p \models_{(\tau,\iota,\varphi)} d \iff T_d(p,c,t)\ge\tau,\;\; I_d(p,c,t)\le\iota,\;\; F_d(p,c,t)\le\varphi.$$

Temporal evolution (one lawful instance). Let $(\otimes,\oplus)$ be a fixed t-norm and s-norm on $[0,1]$ (commutative, associative, monotone; $u\otimes 1=u$, $u\oplus 0=u$). For each (p,d,c,t) suppose supportive, contradictory, and unresolved-uncertainty evidences $e^{+}_{p,d,c}(t)$, $e^{-}_{p,d,c}(t)$, $u_{p,d,c}(t)\in[0,1]$, and parameters $\alpha_d,\beta_d,\gamma_d,\delta_d^T,\delta_d^F,\delta_d^I\in[0,1]$. A discrete-time update that preserves $[0,1]^3$ is

$$\begin{aligned}
T_d(p,c,t{+}1) &= \big((1{-}\delta_d^T)\,T_d(p,c,t)\big)\;\oplus\;\big(\alpha_d\,e^{+}_{p,d,c}(t)\big),\\
F_d(p,c,t{+}1) &= \big((1{-}\delta_d^F)\,F_d(p,c,t)\big)\;\oplus\;\big(\beta_d\,e^{-}_{p,d,c}(t)\big),\\
I_d(p,c,t{+}1) &= \big((1{-}\delta_d^I)\,I_d(p,c,t)\big)\;\oplus\;\big(\gamma_d\,u_{p,d,c}(t)\big).
\end{aligned}$$

Basic properties (checked by inequalities). Assume the probabilistic sum $a\oplus b:=a+b-ab$. Then for every p,d,c,t:

$$0\le T_d(p,c,t),\; I_d(p,c,t),\; F_d(p,c,t)\le 1.$$

Indeed, for T_d: each factor $1-w_{d,q}T_q\in[0,1]$, hence $\prod_q(1-w_{d,q}T_q)\in[0,1]$ and $T_d=1-\prod_q(1-w_{d,q}T_q)\in[0,1]$. For F_d: each $(F_q)^{w_{d,q}}\in[0,1]$ so the product is in $[0,1]$. For I_d: monotonicity of A_I keeps values in $[0,1]$. For the evolution, writing $a=(1{-}\delta)\,X\in[0,1]$ and $b=\eta\,Z\in[0,1]$, we have $a\oplus b=a+b-ab\in[0,1]$ and $\partial(a\oplus b)/\partial b=1-a\ge 0$, so each update stays in $[0,1]$ and is nondecreasing in its evidence input.

Example 3.14 (Remote Team Meeting: Extraversion in a Video Call (Single Context))**. Person** p = Rina, **context** c = "weekly remote team meeting", **time** $t=t_0$. Trait d = Ext (Extraversion). Items and weights (sum to 1):

Q_{Ext} = {voluntary speaking (q_1), initiates group chat (q_2), camera-on engagement (q_3)}, $\quad(w_1,w_2,w_3)=(0.40,0.35,0.25)$.

Neutrosophic item appraisals for (p,c,t_0):

item	T_q	I_q	F_q
q_1	0.70	0.10	0.20
q_2	0.60	0.20	0.30
q_3	0.50	0.30	0.40

Trait-level aggregation (from the NP definition):

$$T_{\mathrm{Ext}}=1-\prod_{q\in Q_{\mathrm{Ext}}}\big(1-w_qT_q\big),\quad I_{\mathrm{Ext}}=\sum_q w_qI_q,\quad F_{\mathrm{Ext}}=\prod_q(F_q)^{w_q}\;.$$

Step-by-step numerics.

$$\begin{aligned}
T_{\mathrm{Ext}} &= 1-\big(1-0.40\cdot0.70\big)\big(1-0.35\cdot0.60\big)\big(1-0.25\cdot0.50\big)\\
&= 1-(0.72)(0.79)(0.875)=1-0.4977=0.5023,\\
I_{\mathrm{Ext}} &= 0.40\cdot0.10+0.35\cdot0.20+0.25\cdot0.30=0.040+0.070+0.075=0.185,\\
F_{\mathrm{Ext}} &= (0.20)^{0.40}(0.30)^{0.35}(0.40)^{0.25}=\exp\Big(0.40\ln0.20+0.35\ln0.30+0.25\ln0.40\Big)\\
&= \exp(-1.294238\ldots)=0.2741.
\end{aligned}$$

Decision. With thresholds $(\tau,\iota,\varphi)=(0.50,0.25,0.50)$,

$$T_{\mathrm{Ext}}=0.5023\ge\tau,\quad I_{\mathrm{Ext}}=0.185\le\iota,\quad F_{\mathrm{Ext}}=0.2741\le\varphi,$$

so Rina *expresses* Extraversion in this meeting context.

Example 3.15 (Office Workday: Conscientiousness while Preparing a Report)**. Person** p = Arun,
context c = "onsite, preparing quarterly report",
time $t = t_0$.

Trait d = Consc (Conscientiousness). Items and weights (sum to 1):

$$Q_{\text{Consc}} = \{\text{plan checklist } (q_1),\ \text{meets micro-deadlines } (q_2),\ \text{keeps clean notes } (q_3),\ \text{final self-review } (q_4)\},$$

$$(w_1, w_2, w_3, w_4) = (0.30, 0.25, 0.25, 0.20).$$

Neutrosophic item appraisals for (p, c, t_0):

item	q_1	q_2	q_3	q_4
T_q	0.80	0.60	0.70	0.50
I_q	0.10	0.25	0.20	0.15
F_q	0.10	0.20	0.15	0.30

Trait-level aggregation:

$$T_{\text{Consc}} = 1 - \prod_q (1 - w_q T_q), \quad I_{\text{Consc}} = \sum_q w_q I_q, \quad F_{\text{Consc}} = \prod_q (F_q)^{w_q}\ .$$

Step-by-step numerics.

$$\begin{aligned} T_{\text{Consc}} &= 1 - \big(1 - 0.30 \cdot 0.80\big)\big(1 - 0.25 \cdot 0.60\big)\big(1 - 0.25 \cdot 0.70\big)\big(1 - 0.20 \cdot 0.50\big) \\ &= 1 - (0.76)(0.85)(0.825)(0.90) = 1 - 0.479655 = 0.520345, \end{aligned}$$

$$\begin{aligned} I_{\text{Consc}} &= 0.30 \cdot 0.10 + 0.25 \cdot 0.25 + 0.25 \cdot 0.20 + 0.20 \cdot 0.15 \\ &= 0.030 + 0.0625 + 0.050 + 0.030 = 0.1725, \end{aligned}$$

$$\begin{aligned} F_{\text{Consc}} &= (0.10)^{0.30}(0.20)^{0.25}(0.15)^{0.25}(0.30)^{0.20} = \exp\Big(0.30 \ln 0.10 + 0.25 \ln 0.20 + 0.25 \ln 0.15 + 0.20 \ln 0.30\Big) \\ &= \exp(-1.808210\ldots) = 0.16395. \end{aligned}$$

Decision. With $(\tau, \iota, \varphi) = (0.52, 0.25, 0.40)$,

$$T_{\text{Consc}} = 0.520345 \geq \tau, \quad I_{\text{Consc}} = 0.1725 \leq \iota, \quad F_{\text{Consc}} = 0.16395 \leq \varphi,$$

so Arun *expresses* Conscientiousness in this office-report context.

3.6 Neutrosophic Personality Trait

A neutrosophic personality trait represents an individual's attribute by independent truth, indeterminacy, and falsity degrees, allowing simultaneous contradictory expressions contextually [81].

Definition 3.16 (Neutrosophic Personality Trait)**.** [81] A *neutrosophic personality trait* is a personality attribute represented by a triad

$$\tau = (T_\tau, I_\tau, F_\tau) \in [0, 1]^3,$$

where T_τ is the degree to which the individual manifests the trait, F_τ is the degree to which the individual manifests the opposite (anti-trait), and I_τ is the degree of indeterminacy/oscillation between the two. The three components are assessed independently and may satisfy $0 \leq T_\tau + I_\tau + F_\tau \leq 3$, so that simultaneous partial trait and partial anti-trait are allowed.

Example 3.17 (Neutrosophic Introversion–Extraversion Trait)**.** Consider the opposite pair (Extravert, Introvert). For a given person at a given period:

$$T_{\text{Extravert}} = 0.65, \quad I_{\text{Extravert}} = 0.20, \quad F_{\text{Extravert}} = 0.40.$$

This means: the person rather acts extravertedly (0.65), sometimes is undecided or context-dependent (0.20), but also shows a non-negligible introvert/anti-extravert side (0.40), e.g. at work meetings the person speaks a lot, but in family gatherings becomes reserved. Because components are independent, $0.65 + 0.20 + 0.40 = 1.25 > 1$ is admissible and captures the psychological mixture.

3.7 Neutrosophic Group Trait

A neutrosophic group trait evaluates collective behavior using truth, indeterminacy, and falsity components, capturing internal disagreement, diversity, and situational variability [81].

Definition 3.18 (Neutrosophic Group Trait)**.** [81] Let G be a social, professional, or cultural group. A *neutrosophic group trait* is a mapping

$$\Theta_G : \mathcal{P}(G) \longrightarrow [0,1]^3, \qquad X \mapsto \Theta_G(X) = \big(T_G(X), I_G(X), F_G(X)\big),$$

where for a subgroup $X \subseteq G$, $T_G(X)$ is the degree to which X commonly displays a selected social/behavioral trait (e.g. "collective responsibility"), $F_G(X)$ is the degree to which X displays the opposite (e.g. "individualist detachment"), and $I_G(X)$ is the degree of intra-group fluctuation, disagreement, or contextual ambiguity about that trait. It extends neutrosophic personality traits from individuals to group-level, possibly refined, collective personalities.

Example 3.19 (Project Team's Neutrosophic Cooperation Trait)**.** Let G be a 5-person project team. For the subgroup $X = G$ and the trait "cooperative work style" we obtain

$$T_G(X) = 0.72, \quad I_G(X) = 0.18, \quad F_G(X) = 0.35.$$

Interpretation: the team is largely cooperative (0.72), but some tasks raise uncertainty (0.18)—for instance, when deadlines are unclear—and a sizeable minority sometimes acts competitively or non-cooperatively (0.35), e.g. two senior members override others. Since the trait is recorded at group level, T_G and F_G may both be high, reflecting internal diversity.

3.8 Neutrosophic Temperaments

Neutrosophic temperaments model classical temperament types with separate truth, indeterminacy, and falsity levels, enabling mixed, evolving, or paradoxical emotional profiles [81].

Definition 3.20 (Neutrosophic Temperaments)**.** [81] A *neutrosophic temperament* is a 4-component assignment

$$\mathsf{NT} = \big((T_s, I_s, F_s),\ (T_c, I_c, F_c),\ (T_m, I_m, F_m),\ (T_p, I_p, F_p)\big)$$

corresponding to the classical temperaments

$$\{\text{sanguine } (s),\ \text{choleric } (c),\ \text{melancholic } (m),\ \text{phlegmatic } (p)\},$$

where for each temperament $\theta \in \{s, c, m, p\}$, T_θ is the degree the person manifests θ, F_θ is the degree the person manifests the opposite/contradictory mode of θ (e.g. anti-sanguine = withdrawn, joyless), and I_θ is the degree of indeterminacy, blending, or situational switching for that temperament. No component is forced to be 1 or to sum to 1, so mixed or evolving temperaments are expressible.

Example 3.21 (Mixed Neutrosophic Temperament of a Lecturer)**.** A university lecturer behaves vividly in class, is sometimes irritable in administration, tends to worry about research deadlines, and stays calm in family life. We can model this as

$$\begin{aligned}
(T_s, I_s, F_s) &= (0.80,\ 0.05,\ 0.10) && \text{(cheerful, talkative in lectures)},\\
(T_c, I_c, F_c) &= (0.55,\ 0.20,\ 0.30) && \text{(assertive, but not always)},\\
(T_m, I_m, F_m) &= (0.40,\ 0.25,\ 0.35) && \text{(reflective, sometimes anxious)},\\
(T_p, I_p, F_p) &= (0.60,\ 0.10,\ 0.25) && \text{(calm in domestic context)}.
\end{aligned}$$

Because each triple is neutrosophic, the lecturer may at the same time show $T_c = 0.55$ (choleric drive) and $T_p = 0.60$ (phlegmatic calm), which would be impossible in a strictly crisp temperament classification. The indeterminacy parts $(0.05, 0.20, 0.25, 0.10)$ capture context-dependent or unobserved emotional states.

3.9 Neutrosophic Persona

Persona is a role-based public identity individuals present, shaped by context, expectations, impression management, masking authentic traits and intentions strategically [97–100]. Neutrosophic Persona models enacted role descriptors using triplet truth, indeterminacy, falsehood; trait-to-descriptor loadings, masking, embellishment, uncertainty, compositional aggregation and decisions [81].

Definition 3.22 (Neutrosophic Persona (NPer))**.** [81] Let P be a nonempty set of persons, R a nonempty set of *roles/personae*, D a nonempty set of personality dimensions (traits), G a nonempty set of *persona descriptors* (declared attributes, claims, or behaviors), C a set of contexts (situations), and $\mathbb{T}$ a time index (discrete or continuous). Assume an underlying neutrosophic personality appraisal

$$N_D(\cdot, c, t) : P \times D \longrightarrow [0,1]^3, \qquad (p, d) \longmapsto \big(T_d(p,c,t),\ I_d(p,c,t),\ F_d(p,c,t)\big),$$

with independent components in $[0,1]$.

For each role $r \in R$, context $c \in C$, time $t \in \mathbb{T}$, and person $p \in P$, a *neutrosophic persona state* over descriptors G is the map

$$N_G^{(r)}(\cdot\,; p, c, t) : G \to [0,1]^3, \qquad g \mapsto \big(T_g^{(r)}(p,c,t),\ I_g^{(r)}(p,c,t),\ F_g^{(r)}(p,c,t)\big),$$

interpreted as the degrees to which p *enacts* (truth), is *uncertain about* (indeterminacy), or *fails/contradicts* (falsehood) descriptor g when presenting role r at (c,t).

Impression-management operator. Fix weights $\kappa_g(d) \in [0,1]$ mapping traits to descriptors, and a *mask/selectivity* $m_{r,d}(p,c,t) \in [0,1]$ (how much trait d is highlighted by role r). Define the effective loadings $\theta_{r,g}(d;p,c,t) := \kappa_g(d)\, m_{r,d}(p,c,t) \in [0,1]$. Let $f^+_{r,g}(p,c,t)$, $f^-_{r,g}(p,c,t)$, $u_{r,g}(p,c,t) \in [0,1]$ encode, respectively, fabrication/embellishment, denial/counter-signal, and added uncertainty for descriptor g under role r. Let $\oplus$ be an s-norm on $[0,1]$ (commutative, associative, monotone; neutral 0), and let A_I be a monotone aggregator on $[0,1]$ (e.g., weighted mean or max). Then for each $g \in G$ set

$$T_g^{(r)}(p,c,t) \;:=\; \underbrace{1 - \prod_{d \in D} \big(1 - \theta_{r,g}(d;p,c,t)\, T_d(p,c,t)\big)}_{\text{selected-trait synergy}} \;\oplus\; f^+_{r,g}(p,c,t),$$

$$I_g^{(r)}(p,c,t) \;:=\; \mathsf{A}_I\big(\{\,\theta_{r,g}(d;p,c,t)\, I_d(p,c,t) : d \in D\,\}\big) \;\oplus\; u_{r,g}(p,c,t),$$

$$F_g^{(r)}(p,c,t) \;:=\; \underbrace{\prod_{d \in D} \big(F_d(p,c,t)\big)^{\theta_{r,g}(d;p,c,t)}}_{\text{propagated non-expression}} \;\oplus\; f^-_{r,g}(p,c,t).$$

The *neutrosophic persona* of p as r at (c,t) is the vector

$$\mathrm{NPer}(p,r,c,t) \;:=\; \big(N_G^{(r)}(\cdot\,; p,c,t)\big) \;\in\; \big([0,1]^3\big)^G.$$

Decision rule (role-conformity). Given thresholds $(\tau, \iota, \varphi) \in [0,1]^3$, we say that p *credibly enacts* descriptor g in role r at (c,t) iff

$$p \models^{(r)}_{(\tau,\iota,\varphi)} g \quad\Longleftrightarrow\quad T_g^{(r)}(p,c,t) \ge \tau,\ \ I_g^{(r)}(p,c,t) \le \iota,\ \ F_g^{(r)}(p,c,t) \le \varphi.$$

Consistency and bounds (checked by explicit inequalities). Assume the probabilistic sum $a \oplus b := a + b - ab$ as the s-norm. Then for any p, r, c, t and $g \in G$:

$$0 \le T_g^{(r)}(p,c,t),\ I_g^{(r)}(p,c,t),\ F_g^{(r)}(p,c,t) \le 1.$$

Indeed, for $T_g^{(r)}$ the product contains factors $1 - \theta T_d \in [0,1]$, hence $\prod_d (1 - \theta T_d) \in [0,1]$ and $1 - \prod_d (1 - \theta T_d) \in [0,1]$; applying $a \oplus b = a + b - ab \in [0,1]$ keeps the result in $[0,1]$. The same argument shows $F_g^{(r)} \in [0,1]$ since each $(F_d)^\theta \in [0,1]$. For $I_g^{(r)}$, monotonicity of A_I and the fact that each input lies in $[0,1]$ imply the bound.

Monotonicity in mask/fabrication. Fix all inputs but $m_{r,d}$ and $f^+_{r,g}$. Writing

$$S_T \;:=\; 1 - \prod_d \big(1 - \kappa_g(d)\, m_{r,d}\, T_d\big),$$

we have $\partial S_T / \partial m_{r,d} = \kappa_g(d)\, T_d \cdot \prod_{d' \neq d} \big(1 - \kappa_g(d')\, m_{r,d'}\, T_{d'}\big) \ge 0$. Moreover, with probabilistic sum $T_g^{(r)} = S_T \oplus f^+$,

$$\frac{\partial (S_T \oplus f^+)}{\partial f^+} = \frac{\partial (S_T + f^+ - S_T f^+)}{\partial f^+} = 1 - S_T \;\ge\; 0.$$

Hence $T_g^{(r)}$ is nondecreasing in each $m_{r,d}$ and in $f^+_{r,g}$. Analogous one-line checks show that $F_g^{(r)}$ is nondecreasing in $f^-_{r,g}$ and each F_d.

Personality-to-persona consistency (identity case). If $m_{r,d} \equiv 1$, $f^+_{r,g} \equiv f^-_{r,g} \equiv u_{r,g} \equiv 0$, and the mapping κ is a projection (e.g., $G = D$ and $\kappa_g(d) = 1[g = d]$), then

$$T_g^{(r)}(p,c,t) = T_g(p,c,t), \qquad I_g^{(r)}(p,c,t) = I_g(p,c,t), \qquad F_g^{(r)}(p,c,t) = F_g(p,c,t),$$

so the persona coincides with the underlying personality on the projected coordinates.

Example 3.23 (Job Interview Persona: "Candidate" Enacting Two Descriptors)**. Person** p = Naomi, **role** r = Candidate, **context** c = "onsite interview", time $t = t_0$. s-norm $\oplus$: probabilistic sum $a \oplus b = a + b - ab$. Indeterminacy aggregator A_I: *weighted mean with weights proportional to* θ.

Underlying neutrosophic personality (selected traits D**).**

d	Ext	Consc	Agree	Stability
T_d	0.65	0.70	0.55	0.60
I_d	0.15	0.10	0.20	0.20
F_d	0.20	0.15	0.25	0.25

Mask (selectivity) for the role r $(m_{r,d})$:

$$m_{\text{Ext}} = 0.90, \quad m_{\text{Consc}} = 0.80, \quad m_{\text{Agree}} = 0.60, \quad m_{\text{Stability}} = 0.80.$$

Descriptor 1: g_1 = "Confident communicator". Loadings κ_{g_1}: $\kappa(\text{Ext}) = 0.60$, $\kappa(\text{Stability}) = 0.30$, $\kappa(\text{Consc}) = 0.10$, $\kappa(\text{Agree}) = 0$.

$$\theta = \kappa \odot m: \quad \theta_{\text{Ext}} = 0.54,\ \theta_{\text{Stability}} = 0.24,\ \theta_{\text{Consc}} = 0.08,\ \theta_{\text{Agree}} = 0.$$

Impression signals: $f^+ = 0.05$, $f^- = 0.03$, $u = 0.10$.

Truth (selected-trait synergy $\oplus$ *fabrication).*

$$\begin{aligned} 1 - \prod_d (1 - \theta_d T_d) &= 1 - (1 - 0.54 \cdot 0.65)(1 - 0.24 \cdot 0.60)(1 - 0.08 \cdot 0.70) \\ &= 1 - (0.649)(0.856)(0.944) = 1 - 0.524433536 = 0.475566464, \\ T_{g_1}^{(r)} &= 0.475566464 \ \oplus\ 0.05 = 0.475566464 + 0.05 - 0.0237783232 = 0.5017881408. \end{aligned}$$

Indeterminacy (weighted mean $\to \oplus u$*).*

$$\sum \theta = 0.54 + 0.24 + 0.08 = 0.86, \quad \frac{\sum \theta_d I_d}{\sum \theta} = \frac{0.54 \cdot 0.15 + 0.24 \cdot 0.20 + 0.08 \cdot 0.10}{0.86} = \frac{0.137}{0.86} = 0.159302\ldots$$

$$I_{g_1}^{(r)} = 0.159302 \ \oplus\ 0.10 = 0.159302 + 0.10 - 0.0159302 = 0.243372.$$

Falsehood (propagated non-expression $\oplus f^-$*).*

$$\prod_d (F_d)^{\theta_d} = (0.20)^{0.54}(0.25)^{0.24}(0.15)^{0.08} = \exp\big(0.54 \ln 0.20 + 0.24 \ln 0.25 + 0.08 \ln 0.15\big) \approx 0.2587,$$

$$F_{g_1}^{(r)} = 0.2587 \ \oplus\ 0.03 = 0.2587 + 0.03 - 0.007761 \approx 0.2810.$$

Descriptor 2: g_2 = "Detail-oriented professional". Loadings κ_{g_2}: $\kappa(\text{Consc}) = 0.70$, $\kappa(\text{Agree}) = 0.20$, $\kappa(\text{Ext}) = 0.10$, $\kappa(\text{Stability}) = 0$.

$$\theta:\ \theta_{\text{Consc}} = 0.56,\ \theta_{\text{Agree}} = 0.12,\ \theta_{\text{Ext}} = 0.09. \quad (f^+, f^-, u) = (0.02, 0.04, 0.08).$$

$$\begin{aligned} 1 - \prod (1 - \theta T) &= 1 - (1 - 0.56 \cdot 0.70)(1 - 0.12 \cdot 0.55)(1 - 0.09 \cdot 0.65) \\ &= 1 - (0.608)(0.934)(0.9415) = 1 - 0.534651488 = 0.465348512, \\ T_{g_2}^{(r)} &= 0.465348512 \ \oplus\ 0.02 = 0.465348512 + 0.02 - 0.009306970 = 0.476041542. \end{aligned}$$

$$\sum \theta = 0.77, \quad \frac{\sum \theta I}{\sum \theta} = \frac{0.56 \cdot 0.10 + 0.12 \cdot 0.20 + 0.09 \cdot 0.15}{0.77} = \frac{0.0935}{0.77} = 0.1214286,$$

$$I_{g_2}^{(r)} = 0.1214286 \ \oplus\ 0.08 = 0.1214286 + 0.08 - 0.0097143 = 0.1917143.$$

$$\prod (F)^{\theta} = (0.15)^{0.56}(0.25)^{0.12}(0.20)^{0.09} = \exp(-1.37359) \approx 0.2529, \quad F_{g_2}^{(r)} = 0.2529 \ \oplus\ 0.04 \approx 0.2828.$$

Decision (example thresholds). For g_1: $(\tau, \iota, \varphi) = (0.50, 0.30, 0.50) \Rightarrow (T, I, F) = (0.502, 0.243, 0.281) \Rightarrow$ *credible*. For g_2: $(\tau, \iota, \varphi) = (0.47, 0.25, 0.50) \Rightarrow (0.476, 0.192, 0.283) \Rightarrow$ *credible*.

Example 3.24 (Online Dating Profile Persona: "Profile Author" with Two Descriptors)**. Person** p = Leo, **role** r = ProfileAuthor, **context** c = "create dating profile", time $t = t_0$. $\oplus$ is the probabilistic sum; A_I is the θ*-weighted mean*.

Underlying personality (selected D**).**

d	Open	Ext	Consc	Agree
T_d	0.55	0.50	0.60	0.65
I_d	0.25	0.25	0.20	0.15
F_d	0.30	0.35	0.25	0.20

Mask m **for the role.**

$$m_{\text{Open}} = 0.90, \quad m_{\text{Ext}} = 0.80, \quad m_{\text{Consc}} = 0.50, \quad m_{\text{Agree}} = 0.70.$$

Descriptor 1: g_1 = "Adventurous traveler". $\kappa(\text{Open}, \text{Ext}, \text{Agree}, \text{Consc}) = (0.70, 0.25, 0.05, 0)$.

$$\theta: \ \theta_{\text{Open}} = 0.63, \ \theta_{\text{Ext}} = 0.20, \ \theta_{\text{Agree}} = 0.035. \quad (f^+, f^-, u) = (0.10, 0.02, 0.15).$$

Truth.

$$\begin{aligned} 1 - \prod (1 - \theta T) &= 1 - (1 - 0.63 \cdot 0.55)(1 - 0.20 \cdot 0.50)(1 - 0.035 \cdot 0.65) \\ &= 1 - (0.6535)(0.90)(0.97725) = 1 - 0.5747695875 = 0.4252304125, \\ T_{g_1}^{(r)} &= 0.4252304125 \ \oplus \ 0.10 = 0.4252304125 + 0.10 - 0.0425230413 = 0.4827073712. \end{aligned}$$

Indeterminacy.

$$\sum \theta = 0.865, \quad \frac{\sum \theta I}{\sum \theta} = \frac{0.63 \cdot 0.25 + 0.20 \cdot 0.25 + 0.035 \cdot 0.15}{0.865} = \frac{0.21275}{0.865} = 0.245665\ldots$$

$$I_{g_1}^{(r)} = 0.245665 \ \oplus \ 0.15 = 0.245665 + 0.15 - 0.0368498 = 0.358815.$$

Falsehood.

$$\prod (F)^{\theta} = (0.30)^{0.63}(0.35)^{0.20}(0.20)^{0.035} = \exp\big(0.63 \ln 0.30 + 0.20 \ln 0.35 + 0.035 \ln 0.20\big) \approx 0.3590,$$

$$F_{g_1}^{(r)} = 0.3590 \ \oplus \ 0.02 = 0.3590 + 0.02 - 0.00718 = 0.3718.$$

Decision (e.g., $(\tau, \iota, \varphi) = (0.48, 0.40, 0.50)$*):* $(T, I, F) = (0.483, 0.359, 0.372) \Rightarrow$ *credible*.

Descriptor 2: g_2 = "Responsible partner". $\kappa(\text{Consc}, \text{Agree}, \text{Ext}, \text{Open}) = (0.60, 0.30, 0.10, 0)$.

$$\theta: \ \theta_{\text{Consc}} = 0.30, \ \theta_{\text{Agree}} = 0.21, \ \theta_{\text{Ext}} = 0.08. \quad (f^+, f^-, u) = (0.03, 0.01, 0.10).$$

$$\begin{aligned} 1 - \prod (1 - \theta T) &= 1 - (1 - 0.30 \cdot 0.60)(1 - 0.21 \cdot 0.65)(1 - 0.08 \cdot 0.50) \\ &= 1 - (0.82)(0.8635)(0.96) = 1 - 0.6797472 = 0.3202528, \\ T_{g_2}^{(r)} &= 0.3202528 \ \oplus \ 0.03 = 0.3202528 + 0.03 - 0.0096076 = 0.3406452. \end{aligned}$$

$$\sum \theta = 0.59, \quad \frac{\sum \theta I}{\sum \theta} = \frac{0.30 \cdot 0.20 + 0.21 \cdot 0.15 + 0.08 \cdot 0.25}{0.59} = \frac{0.1115}{0.59} = 0.188983,$$

$$I_{g_2}^{(r)} = 0.188983 \ \oplus \ 0.10 = 0.188983 + 0.10 - 0.018898 = 0.270085.$$

$$\prod (F)^{\theta} = (0.25)^{0.30}(0.20)^{0.21}(0.35)^{0.08} = \exp(-0.837856) \approx 0.4328, \quad F_{g_2}^{(r)} = 0.4328 \ \oplus \ 0.01 = 0.4385.$$

Decision (e.g., $(\tau, \iota, \varphi) = (0.35, 0.35, 0.60)$*):* $T = 0.341 < \tau$ (fail), $I = 0.270 \le 0.35$, $F = 0.439 \le 0.60 \Rightarrow$ *not yet credible*. This shows how mask/selectivity and light embellishment shape a public persona, while the neutrosophic triplet keeps truth, uncertainty, and contradiction explicit.

3.10 Neutrosophic Negotiation

Classical negotiation theory investigates how two or more actors move from divergent positions to an acceptable settlement, paying attention to interests, available options, and feasible outcomes [101–105]. Within this literature, two reference notions are repeatedly used: the *Best Alternative to a Negotiated Agreement* (BATNA) and the *Zone of Possible Agreement* (ZOPA). Roughly speaking, the BATNA of a party is the value that the party can secure without the current negotiation, i.e. the payoff of the best outside opportunity [106–110]. The ZOPA, in turn, is the subset of outcomes for which both parties obtain at least their own BATNA, so that a mutually acceptable agreement is attainable; when this interval is empty, no rational deal can be struck [111–114].

Definition 3.25 (Neutrosophic Best Alternative to a Negotiated Agreement (Neutrosophic BATNA))**.** Let N represent a negotiation between two parties A (Agent 1) and B (Agent 2), where the set of all possible deals is $\mathcal{D} \subseteq \mathbb{R}^2$. The *Neutrosophic Best Alternative to a Negotiated Agreement (Neutrosophic BATNA)* incorporates the degrees of truth (T), indeterminacy (I), and falsity (F) into the evaluation of alternatives.

Formally, the Neutrosophic BATNA for each party $i \in \{A, B\}$ is defined as:

$$\text{NBATNA}_i = \max_{\alpha \in \mathcal{A}_i} \mathcal{N}_i(\alpha), \quad \mathcal{N}_i(\alpha) = (T_i(\alpha), I_i(\alpha), F_i(\alpha)),$$

where:

- $\mathcal{A}_i$: The set of alternatives available to party i outside the current negotiation N (e.g., fallback options, external agreements).
- $\mathcal{N}_i : \mathcal{A}_i \to [0, 1]^3$: The neutrosophic utility function of party i, mapping each alternative α to a tuple:

  $$\mathcal{N}_i(\alpha) = (T_i(\alpha), I_i(\alpha), F_i(\alpha)),$$

  where:

  $$T_i(\alpha) + I_i(\alpha) + F_i(\alpha) \leq 1, \quad T_i, I_i, F_i \in [0, 1].$$

- NBATNA_i: The maximum neutrosophic utility for party i, which quantifies the best outcome they can achieve independently.

Acceptance Condition. For any negotiated deal $d \in \mathcal{D}$, party i will only accept d if:

$$\mathcal{N}_i(d) \succeq \text{NBATNA}_i,$$

where $\succeq$ denotes a partial order such that:

$$(T_i(d), I_i(d), F_i(d)) \succeq (T_i, I_i, F_i) \iff T_i(d) \geq T_i,\ I_i(d) \leq I_i, \text{ and } F_i(d) \leq F_i.$$

Remark 3.26 (Neutrosophic BATNA)**.** Fuzzy BATNA is a special case of Neutrosophic BATNA where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic BATNA is notable for its ability to generalize both Neutrosophic and Fuzzy BATNA.

Example 3.27 (Neutrosophic BATNA in a salary renegotiation)**.** A software engineer (party A) is negotiating with her current company (party B) for the next fiscal year. The engineer currently earns 6.0M JPY/year. She has received a verbal indication from another company (Company C) that they are "very likely" to offer 6.5M JPY/year, but the offer is not yet in writing and the start date is not fully fixed. From the engineer's point of view, this outside option is an *uncertain* alternative and must be modeled neutrosophically.

Let the set of outside alternatives for A be

$$\mathcal{A}_A = \{\alpha_1 = \text{stay at 6.0M},\ \alpha_2 = \text{verbal 6.5M from C}\}.$$

Define the neutrosophic utility of staying at 6.0M as

$$\mathcal{N}_A(\alpha_1) = (T_A(\alpha_1), I_A(\alpha_1), F_A(\alpha_1)) = (0.80, 0.05, 0.05),$$

meaning: she is almost fully sure she can stay (truth 0.80), there is a small scheduling/role uncertainty (0.05), and almost no risk it collapses (0.05).

For the verbal 6.5M JPY offer from C, she evaluates:

$$\mathcal{N}_A(\alpha_2) = (0.65, 0.25, 0.10),$$

i.e. truth 0.65 (HR said "very likely"), indeterminacy 0.25 (they are waiting for budget approval), and falsity 0.10 (there is still a chance of rejection).

To determine her *Neutrosophic BATNA*, A compares these two alternatives with the partial order

$$(T, I, F) \succeq (T', I', F') \iff T \geq T',\ I \leq I',\ F \leq F'.$$

Although α_2 has better pay, it also has higher indeterminacy and falsity. Suppose A prioritizes *certainty over amount*, so she selects

$$\text{NBATNA}_A = \mathcal{N}_A(\alpha_1) = (0.80, 0.05, 0.05).$$

Now the company B makes a counter–proposal $d \in \mathcal{D}$: "6.3M JPY, fixed role, but possible relocation within the year." A evaluates this negotiated deal as

$$\mathcal{N}_A(d) = (0.83, 0.10, 0.02),$$

because the salary is higher than 6.0M (truth increases to 0.83), the relocation introduces some indeterminacy (0.10), but the falsity is low (0.02) since the company is stable.

Check acceptance:

$$(0.83, 0.10, 0.02) \succeq (0.80, 0.05, 0.05)$$

fails on indeterminacy since $0.10 > 0.05$, but A may allow a tolerance band

$$I_A(d) \leq I_A(\text{NBATNA}) + \varepsilon, \quad \varepsilon = 0.05,$$

then $0.10 \leq 0.05 + 0.05$ holds. Hence, with a small tolerance, the neutrosophic BATNA is satisfied and A can rationally accept d.

Definition 3.28 (Neutrosophic Zone of Possible Agreement (Neutrosophic ZOPA))**.** The *Neutrosophic Zone of Possible Agreement (Neutrosophic ZOPA)* is the set of feasible deals where the neutrosophic utility of both parties meets or exceeds their respective Neutrosophic BATNAs.

Let $\mathcal{N}_A : \mathcal{D} \to [0,1]^3$ and $\mathcal{N}_B : \mathcal{D} \to [0,1]^3$ represent the neutrosophic utility functions of parties A and B, respectively. Then the Neutrosophic ZOPA is defined as:

$$\text{NZOPA} = \{d \in \mathcal{D} \mid \mathcal{N}_A(d) \succeq \text{NBATNA}_A \text{ and } \mathcal{N}_B(d) \succeq \text{NBATNA}_B\}.$$

Existence Condition. The Neutrosophic ZOPA exists if and only if there exists a deal $d \in \mathcal{D}$ such that:

$$\mathcal{N}_A(d) \succeq \text{NBATNA}_A \quad \text{and} \quad \mathcal{N}_B(d) \succeq \text{NBATNA}_B.$$

If no such deal d exists, the negotiation is said to have a *Neutrosophic Negative Bargaining Zone (NNBZ)*.

Remark 3.29 (Neutrosophic ZOPA)**.** Fuzzy ZOPA is a special case of Neutrosophic ZOPA where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic ZOPA is notable for its ability to generalize both Neutrosophic and Fuzzy ZOPA.

Example 3.30 (Neutrosophic ZOPA in an international supply contract)**.** A Japanese retailer (buyer, party A) wants to secure a yearly supply of 10,000 units of an electronic component from a foreign manufacturer (seller, party B). Both sides face volatile shipping costs and regulatory delays, so every candidate deal has truth, indeterminacy, and falsity parts.

Let a deal be

$$d = (\text{unit price, delivery window}) \in \mathcal{D}.$$

The buyer A has an outside arrangement with a domestic distributor: 10,000 units at 1,000 JPY each, but only 80% delivery certainty because of domestic shortages. A evaluates this fallback as

$$\text{NBATNA}_A = (0.78, 0.15, 0.07),$$

i.e. about 78% sure to get the goods, 15% unclear, 7% chance of disruption.

The seller B has an alternative client in another country who is willing to buy at a slightly lower volume but with slower payment. B evaluates that alternative as

$$\text{NBATNA}_B = (0.70, 0.20, 0.10).$$

Now consider three candidate deals in $\mathcal{D}$:

1) d_1: 10,000 units, 950 JPY/unit, delivery in 30 days, standard shipping. Buyer's evaluation:

$$\mathcal{N}_A(d_1) = (0.82, 0.12, 0.06),$$

Seller's evaluation:

$$\mathcal{N}_B(d_1) = (0.74, 0.18, 0.08).$$

We check:

$$\mathcal{N}_A(d_1) \succeq \text{NBATNA}_A \quad \text{and} \quad \mathcal{N}_B(d_1) \succeq \text{NBATNA}_B.$$

For A: $0.82 \geq 0.78$, $0.12 \leq 0.15$, $0.06 \leq 0.07 \Rightarrow$ accepted. For B: $0.74 \geq 0.70$, $0.18 \leq 0.20$, $0.08 \leq 0.10 \Rightarrow$ accepted. Hence $d_1 \in$ NZOPA.

2) d_2: 10,000 units, 900 JPY/unit, delivery in 15 days, air freight. Buyer's evaluation (faster, cheaper, but customs risk):

$$\mathcal{N}_A(d_2) = (0.85, 0.20, 0.10),$$

Seller's evaluation (tighter schedule):

$$\mathcal{N}_B(d_2) = (0.68, 0.25, 0.12).$$

Here $\mathcal{N}_A(d_2) \succeq \text{NBATNA}_A$ holds on T but fails on I and F unless A allows a higher uncertainty band. For B, $0.68 < 0.70$ and $0.25 > 0.20$, so without relaxation $d_2 \notin$ NZOPA.

3) d_3: 9,500 units, 960 JPY/unit, delivery in 30–40 days, mixed freight. Buyer's evaluation:

$$\mathcal{N}_A(d_3) = (0.80, 0.14, 0.06),$$

Seller's evaluation:

$$\mathcal{N}_B(d_3) = (0.73, 0.19, 0.08).$$

Both satisfy their NBATNAs, so $d_3 \in$ NZOPA.

Therefore, in this real–world supply negotiation we obtain

$$\text{NZOPA} = \{d_1, d_3\},$$

while d_2 lies in a *neutrosophic negative bargaining zone* unless at least one party is willing to accept higher indeterminacy or falsity. This illustrates how the neutrosophic ZOPA retains the classical "overlap of acceptable deals" idea, but also records explicitly how much uncertainty and potential failure each party is ready to tolerate.

3.11 Neutrosophic Framing

Framing is the presentation of identical information in different ways, influencing decision-making behavior by altering perception of outcomes and choices.

Definition 3.31 (Neutrosophic Framing)**.** *Neutrosophic Framing* is a mathematical representation of a decision problem where uncertainty, ambiguity, and contradiction are explicitly incorporated into the evaluation of outcomes. A Neutrosophic frame F_N is defined as:

$$F_N = (A, O, P, V_N, U_N),$$

where:

- $A = \{a_1, a_2, \ldots, a_n\}$: The set of available *actions* or *choices*.
- $O = \{o_1, o_2, \ldots, o_m\}$: The set of possible *outcomes*.
- $P : A \times O \to [0, 1]$: The *probability function*, which assigns a probability $P(o_j|a_i)$ to each outcome $o_j \in O$ given action $a_i \in A$. The function satisfies:
$$\forall a_i \in A, \sum_{o_j \in O} P(o_j|a_i) = 1.$$
- $V_N : O \to [0, 1]^3$: The *neutrosophic valuation function*, which assigns a triple $V_N(o_j) = (T_{o_j}, I_{o_j}, F_{o_j})$ to each outcome $o_j \in O$, where:
  - T_{o_j}: The degree of truth (positive evaluation) of the outcome o_j.
  - I_{o_j}: The degree of indeterminacy (uncertainty or ambiguity) of the outcome o_j.
  - F_{o_j}: The degree of falsity (negative evaluation) of the outcome o_j.

– $T_{o_j} + I_{o_j} + F_{o_j} \leq 1$: Consistency condition ensuring the total evaluation remains bounded.

- $U_N : A \to [0,1]^3$: The *neutrosophic utility function*, defined for each action $a_i \in A$ as:

$$U_N(a_i) = \left(T_{a_i}, I_{a_i}, F_{a_i}\right),$$

where:

$$T_{a_i} = \sum_{o_j \in O} P(o_j|a_i) \cdot T_{o_j}, \quad I_{a_i} = \sum_{o_j \in O} P(o_j|a_i) \cdot I_{o_j}, \quad F_{a_i} = \sum_{o_j \in O} P(o_j|a_i) \cdot F_{o_j}.$$

The decision-maker selects the action $a^* \in A$ that maximizes the *truth utility* T_{a_i} while considering the indeterminacy I_{a_i} and falsity F_{a_i}:

$$a^* = \arg\max_{a_i \in A} T_{a_i}.$$

Remark 3.32 (Neutrosophic Valuation and Decision-Making)**.** Neutrosophic framing allows for a richer evaluation of decision problems by incorporating:

- Positive outcomes (T) that contribute directly to utility.
- Uncertain or ambiguous outcomes (I), which reflect incomplete or unclear information.
- Negative outcomes (F) that reflect losses or contradictions.

This framework can model real-world scenarios where outcomes are not purely true or false but lie within a range of truth, uncertainty, and falsity.

Remark 3.33 (Neutrosophic framing)**.** Fuzzy framing is a special case of Neutrosophic framing where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic framing is notable for its ability to generalize both Neutrosophic and Fuzzy framing.

Example 3.34 (Neutrosophic framing for choosing a work arrangement)**.** A data analyst must choose a workstyle for the next quarter. The company offers three actions

$$A = \{a_1 = \text{fully onsite},\ a_2 = \text{hybrid (3 days)},\ a_3 = \text{fully remote}\}.$$

The analyst evaluates each action under three possible outcomes:

$$O = \{o_1 = \text{productivity high},\ o_2 = \text{productivity moderate},\ o_3 = \text{productivity low due to disruptions}\}.$$

Because of changing project priorities and uncertain family duties (small children, occasional care work), the analyst cannot assign only crisp utilities; instead, she uses a neutrosophic valuation.

First, she specifies subjective outcome probabilities (they sum to 1 for each action):

	$P(o_1\|a_i)$	$P(o_2\|a_i)$	$P(o_3\|a_i)$
a_1 (onsite)	0.55	0.30	0.15
a_2 (hybrid)	0.50	0.35	0.15
a_3 (remote)	0.45	0.30	0.25

Next, she assigns neutrosophic valuations to outcomes. A highly productive week at the office is almost certainly good, but some ambiguity remains (boss may change priorities); moderate productivity is less clearly good; low productivity clearly hurts.

$$V_N(o_1) = (T_{o_1}, I_{o_1}, F_{o_1}) = (0.85,\ 0.10,\ 0.05),$$
$$V_N(o_2) = (T_{o_2}, I_{o_2}, F_{o_2}) = (0.55,\ 0.25,\ 0.10),$$
$$V_N(o_3) = (T_{o_3}, I_{o_3}, F_{o_3}) = (0.15,\ 0.25,\ 0.50).$$

Using the definition of neutrosophic utility

$$U_N(a_i) = \left(T_{a_i}, I_{a_i}, F_{a_i}\right) = \sum_j P(o_j|a_i) \cdot V_N(o_j),$$

we obtain:

1) For a_1 (onsite):

$$\begin{aligned} T_{a_1} &= 0.55(0.85) + 0.30(0.55) + 0.15(0.15) \\ &= 0.4675 + 0.165 + 0.0225 = 0.655, \\ I_{a_1} &= 0.55(0.10) + 0.30(0.25) + 0.15(0.25) \\ &= 0.055 + 0.075 + 0.0375 = 0.1675, \\ F_{a_1} &= 0.55(0.05) + 0.30(0.10) + 0.15(0.50) \\ &= 0.0275 + 0.03 + 0.075 = 0.1325. \end{aligned}$$

2) For a_2 (hybrid):

$$\begin{aligned} T_{a_2} &= 0.50(0.85) + 0.35(0.55) + 0.15(0.15) \\ &= 0.425 + 0.1925 + 0.0225 = 0.64, \\ I_{a_2} &= 0.50(0.10) + 0.35(0.25) + 0.15(0.25) \\ &= 0.05 + 0.0875 + 0.0375 = 0.175, \\ F_{a_2} &= 0.50(0.05) + 0.35(0.10) + 0.15(0.50) \\ &= 0.025 + 0.035 + 0.075 = 0.135. \end{aligned}$$

3) For a_3 (remote):

$$\begin{aligned} T_{a_3} &= 0.45(0.85) + 0.30(0.55) + 0.25(0.15) \\ &= 0.3825 + 0.165 + 0.0375 = 0.585, \\ I_{a_3} &= 0.45(0.10) + 0.30(0.25) + 0.25(0.25) \\ &= 0.045 + 0.075 + 0.0625 = 0.1825, \\ F_{a_3} &= 0.45(0.05) + 0.30(0.10) + 0.25(0.50) \\ &= 0.0225 + 0.03 + 0.125 = 0.1775. \end{aligned}$$

Thus the neutrosophic utilities are

$$U_N(a_1) = (0.655,\ 0.1675,\ 0.1325), \quad U_N(a_2) = (0.64,\ 0.175,\ 0.135), \quad U_N(a_3) = (0.585,\ 0.1825,\ 0.1775).$$

If the decision maker applies the usual rule

$$a^* = \arg\max_{a_i} T_{a_i} \quad \text{subject to} \quad I_{a_i}, F_{a_i} \text{ acceptable},$$

then a_1 (fully onsite) is selected, because it has the largest truth score and reasonably small falsity. However, if the analyst wants to *cap* indeterminacy at 0.17, she might prefer the hybrid option a_2, whose truth is close (0.64) but whose ambiguity is still within tolerance. This shows how neutrosophic framing makes the tradeoff among "good," "uncertain," and "bad" parts explicit, instead of hiding them in a single scalar utility.

3.12 Neutrosophic Mentoring

Mentoring is a structured process where an experienced mentor guides, supports, and transfers knowledge to a less experienced protégé for skill and personal development [115–118].

Definition 3.35 (Neutrosophic Mentoring)**.** *Neutrosophic Mentoring* extends traditional mentoring by incorporating uncertainty, indeterminacy, and falsity into the knowledge transfer process. It is defined as a tuple:

$$M_N = (E, P^N, K, T, G),$$

where:

- $E = \{e_1, e_2\}$: A set of agents where e_1 is the mentor (knowledge provider) and e_2 is the protege (knowledge receiver), with $e_1 \neq e_2$.
- $K = \{k_1, k_2, \ldots, k_n\}$: A finite set of knowledge components shared in the mentoring process.
- $P^N : E \times K \times T \to [0,1]^3$: The *neutrosophic knowledge transfer function*, where $P^N(e_1, k_i, t) = (T_{k_i}, I_{k_i}, F_{k_i})$ represents the truth (T), indeterminacy (I), and falsity (F) degrees of knowledge k_i transferred at time t.
- $T = \{t_0, t_1, \ldots, t_m\}$: A finite or infinite set of discrete or continuous time steps during which mentoring occurs.

- $G : K \to \mathbb{R}^+$: The *goal attainment function*, mapping knowledge k_i to a measurable value g_i, representing the protege's learning progress.

The total neutrosophic knowledge gained by the protege e_2 at time t_m is:

$$K^N_{\text{gain}}(e_2, t_m) = \int_{t_0}^{t_m} \sum_{k_i \in K} (T_{k_i} - F_{k_i}) \cdot G(k_i)\, dt.$$

The mentoring process is considered successful if:

$$K^N_{\text{gain}}(e_2, t_m) \geq K^N_{\text{target}},$$

where K^N_{target} is a predefined neutrosophic learning threshold.

Remark 3.36. Fuzzy Mentoring is a special case of Neutrosophic Mentoring where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Mentoring is notable for its ability to generalize both Neutrosophic and Fuzzy Mentoring.

Example 3.37 (Neutrosophic Mentoring in a software engineering team)**.** Consider a mid–size IT company where a senior engineer e_1 (mentor) is assigned to guide a junior engineer e_2 (protégé) during a 3–month onboarding period. Let

$$K = \{k_1 = \text{Git workflow},\ k_2 = \text{Microservice architecture},\ k_3 = \text{Code review standards},\ k_4 = \text{Incident response}\}$$

be the four knowledge components to be transferred. Mentoring sessions take place weekly, so

$$T = \{t_1,\ t_2,\ \ldots,\ t_{12}\}$$

denotes 12 weeks.

Because the project environment changes (hotfixes, priority changes, production incidents), not every piece of knowledge the mentor conveys is immediately valid or fully understood by the protégé. Hence, each transfer is recorded as a neutrosophic triple

$$P^N(e_1, k_i, t_j) = (T_{i,j},\ I_{i,j},\ F_{i,j}) \in [0, 1]^3.$$

A realistic assessment after week 4 could be:

$P^N(e_1, k_1, t_4) = (0.90, 0.05, 0.05)$ (Git workflow was demonstrated and practiced repeatedly),

$P^N(e_1, k_2, t_4) = (0.55, 0.30, 0.15)$ (microservice principles explained, but current service depends on legacy APIs),

$P^N(e_1, k_3, t_4) = (0.80, 0.10, 0.10)$ (review checklist is stable, only small ambiguity),

$P^N(e_1, k_4, t_4) = (0.40, 0.40, 0.20)$ (incident runbooks changed this week, mentor's guidance partly outdated).

Suppose the company values the four topics differently, e.g.

$$G(k_1) = 1.0, \quad G(k_2) = 1.4, \quad G(k_3) = 0.9, \quad G(k_4) = 1.2,$$

because microservice understanding (k_2) is most critical for long–term productivity, and incident response (k_4) is second.

Using the mentoring gain expression at t_4

$$K^N_{\text{gain}}(e_2, t_4) = \sum_{i=1}^{4} (T_{i,4} - F_{i,4}) \cdot G(k_i),$$

we obtain

$$\begin{aligned} K^N_{\text{gain}}(e_2, t_4) &= (0.90 - 0.05) \cdot 1.0 + (0.55 - 0.15) \cdot 1.4 + (0.80 - 0.10) \cdot 0.9 + (0.40 - 0.20) \cdot 1.2 \\ &= 0.85 \cdot 1.0 + 0.40 \cdot 1.4 + 0.70 \cdot 0.9 + 0.20 \cdot 1.2 \\ &= 0.85 + 0.56 + 0.63 + 0.24 \\ &= 2.28. \end{aligned}$$

Assume the HR department set a neutrosophic mentoring target

$$K^N_{\text{target}} = 2.0$$

for the first month. Since $K^N_{\text{gain}}(e_2, t_4) = 2.28 \geq K^N_{\text{target}}$, the mentoring process is judged *successful* even though some parts of the knowledge (notably k_4) were indeterminate or temporarily false due to changing procedures.

This example shows why neutrosophic mentoring is suitable: it does not require every knowledge transfer to be perfectly true; it allows partially valid, context–dependent, or temporarily outdated guidance to be recorded and still to contribute positively, provided the truth part dominates the falsity part for the weighted, important skills.

Chapter 4: Review and Results: Concepts in Social Science

Within Concepts in Social Science, neutrosophic logic has been employed in various ways [119–124]. In this section, we reexamine topics such as the Neutrosophic Complex Dynamic System.

4.1 Neutrosophic Complex Dynamic System

Neutrosophic Complex Dynamic System evolves complex states while tracking truth, indeterminacy, and falsity degrees, combining evidence via t/s-norm updates iteratively [125].

Definition 4.1 (Neutrosophic Complex Dynamic System)**.** [125] Let $\mathcal{U}$ be a universe of discourse and let $\Omega \subseteq \mathcal{U}$ be a (nonempty) *state space*. For a space $S \in \{\Omega,\ C(\Omega)\}$ (where $C(\Omega) := \mathcal{U} \setminus \Omega$), every element $x \in \mathcal{U}$ is annotated by a (neutrosophic) truth–triplet

$$x\big(T_x^S,\ I_x^S,\ F_x^S\big), \qquad T_x^S,\ I_x^S,\ F_x^S \subseteq [0,1],$$

interpreted as degrees (possibly set–valued) of membership, indeterminacy, and non–membership of x with respect to S.

For integers $k \geq 1$ and $\ell \geq 0$, a (time–indexed) *neutrosophic hyperrelationship* is a map

$$\mathcal{R}_{HR}(t) :\ \Omega(t)^{\,k} \times C\big(\Omega(t)\big)^{\,\ell} \ \longrightarrow\ \mathcal{P}([0,1])^3, \qquad (x,y)\ \mapsto\ \big(T_{\mathcal{R}},\ I_{\mathcal{R}},\ F_{\mathcal{R}}\big),$$

assigning a neutrosophic triplet to each k–tuple of inside states x and each ℓ–tuple of outside states y. The interaction is *closed* when $\ell = 0$ and *open* when $\ell \geq 1$.

A *neutrosophic complex dynamic system* is a time–indexed tuple

$$\mathsf{DN} \ :=\ \big(\ \Omega(t),\ X(t),\ \mathcal{R}_{HR}(t),\ \mathcal{H} \subseteq \mathcal{L}\ \big)_{t \geq 0},$$

where:

- $X(t) = \{\, x_i(T_i^{\Omega}, I_i^{\Omega}, F_i^{\Omega}) \mid i = 1, \ldots, n(t) \,\}$ is the (possibly varying) multiset of system elements together with their neutrosophic triplets relative to $\Omega(t)$;
- $\mathcal{R}_{HR}(t)$ belongs to an admissible family $\mathcal{H}$ of neutrosophic hyperrelationships (with $\mathcal{L}$ the class of all such hyperrelationships on $\Omega(t)$);
- the evolution is governed by a (not necessarily linear) *flow*

  $$\Phi :\ [0,\infty) \times \mathcal{S}\ \to\ \mathcal{S}, \qquad \Phi(0,s) = s,\ \ \Phi(t+s,s) = \Phi\big(t, \Phi(s,s)\big),$$

  on the configuration space $\mathcal{S}$ of all triples $(\Omega, X, \mathcal{R}_{HR})$, updating in time the state space Ω, the element–wise triplets (T, I, F), and the hyperrelationships according to the model's laws (e.g., ODEs, agent rules, or stochastic updates).

Equivalently, in discrete time one may specify an update operator

$$F :\ \mathcal{S}\ \to\ \mathcal{S}, \qquad (\Omega_{t+1}, X_{t+1}, \mathcal{R}_{HR,t+1})\ =\ F\big(\Omega_t, X_t, \mathcal{R}_{HR,t}\big),$$

with the same neutrosophic interpretation of states and interactions.

Example 4.2 (Pandemic mitigation with testing uncertainty (open NCDS))**.** *State space.* Let $\mathcal{U}$ be the population and $\Omega(t) \subseteq \mathcal{U}$ the set of individuals who are *currently infectious* at time t. For each person $x \in \mathcal{U}$ we store a neutrosophic triplet relative to $\Omega(t)$:

$$x\big(T_x^{\Omega}(t),\ I_x^{\Omega}(t),\ F_x^{\Omega}(t)\big) \in [0,1]^3,$$

interpreting T as degree of being infectious, I as diagnostic uncertainty, and F as degree of being noninfectious.

Hyperrelationship (open). Let $A(t) = (a_{ij}(t))$ be the contact matrix, $p_i(t) \in [0,1]$ the positive-test evidence for individual i, $r_i(t) \in [0,1]$ the recovery evidence (e.g., days since onset, symptom resolution), and $y(t) \in [0,1]$ an environmental signal (mobility restriction level, seasonality), all exogenous. Define the neutrosophic hyperrelationship $\mathcal{R}_{HR}(t)$ via the map

$$\big(i,\ (j)_{j \in \mathcal{N}(i)},\ y(t)\big)\ \longmapsto\ \big(T_{\mathcal{R},i}(t),\ I_{\mathcal{R},i}(t),\ F_{\mathcal{R},i}(t)\big),$$

where, for fixed gains $\beta, \kappa, \gamma, \eta, \xi, \delta, \nu \in [0,1]$,

$$T_{\mathcal{R},i}(t) = \min\Big\{1,\ \beta \sum_j a_{ij}(t)\, T_j^{\Omega}(t)\,(1 - F_i^{\Omega}(t)) + \kappa\, p_i(t) + \eta\, y(t)\Big\},$$

$$I_{\mathcal{R},i}(t) = \max\Big\{ I_i^{\Omega}(t)(1-\delta),\ \ \xi\big(1 - (p_i(t) + r_i(t))\big)\Big\},$$

$$F_{\mathcal{R},i}(t) = \min\Big\{1,\ \gamma\, r_i(t) + \nu\big(1 - p_i(t)\big)\Big\}.$$

Dynamics (discrete time). With configuration $\mathcal{S}_t = (\Omega(t), X(t), \mathcal{R}_{HR}(t))$, update

$$(T_i^{\Omega}, I_i^{\Omega}, F_i^{\Omega})(t{+}1) \ := \ \Big(T_{\mathcal{R},i}(t),\ I_{\mathcal{R},i}(t),\ F_{\mathcal{R},i}(t)\Big), \qquad i = 1, \ldots, n(t),$$

and set $\Omega(t{+}1) = \{\, i :\ T_i^{\Omega}(t{+}1) > F_i^{\Omega}(t{+}1)\,\}$.

Interpretation. Truth increases through exposure (contacts, tests) and external conditions; falsity increases through recovery and negative evidence; indeterminacy decays when evidence accumulates but grows with missing or conflicting data. The dependence on $y(t)$ makes the system *open*.

Example 4.3 (Renewable-rich power grid situational awareness (open/closed NCDS))**.** *State space.* Let $\mathcal{U}$ be the set of grid assets (generators G, loads L, lines $\mathcal{E}$, buses $\mathcal{B}$) and exogenous factors (wind, solar, temperature). Define the *secure-operation set* $\Omega(t) \subseteq \mathcal{U}$ as assets operating within limits at time t. Each asset $x \in \mathcal{U}$ carries a triplet $x(T_x^{\Omega}(t), I_x^{\Omega}(t), F_x^{\Omega}(t))$ within $[0,1]^3$.

Asset-level evidences. Let $m_x(t) \in [0,1]$ be a normalized security margin for x (e.g., thermal headroom for a line, voltage margin at a bus), $w_x(t) \in [0,1]$ a data-quality index (inverse of measurement variance / sensor agreement), and $a_x(t) \in [0,1]$ an alarm/severity score when limits are violated.

Hyperrelationship (coupled by network flows). Let $z(t)$ collect the AC power-flow variables and dispatch setpoints. Given the network model, define for each asset x:

$$T_{\mathcal{R},x}(t) = \varphi\big(m_x(t)\big), \qquad F_{\mathcal{R},x}(t) = \varphi\big(a_x(t)\big),$$
$$I_{\mathcal{R},x}(t) = \psi\big(1 - w_x(t)\big),$$

where $\varphi : [0,1] \to [0,1]$ and $\psi : [0,1] \to [0,1]$ are increasing (e.g., $\varphi(u) = u$, $\psi(u) = \min\{1, \lambda u\}$). The triplets of a line $e = (i,j) \in \mathcal{E}$ depend on bus voltages/angles, hence on neighboring assets; renewables inject uncertainty via $w_x(t)$ and alter margins $m_x(t)$.

Dynamics. In continuous time, let the internal grid state $z(t)$ evolve as

$$\dot{z}(t) = f\big(z(t),\ u(t),\ y(t)\big),$$

where $u(t)$ is the operator control (dispatch, topology) and $y(t)$ collects exogenous processes (wind speed, irradiance, ambient temperature). Then set

$$\big(T_x^{\Omega}, I_x^{\Omega}, F_x^{\Omega}\big)(t) \ := \ \big(T_{\mathcal{R},x}(t),\ I_{\mathcal{R},x}(t),\ F_{\mathcal{R},x}(t)\big),$$

and define $\Omega(t) = \{\, x :\ T_x^{\Omega}(t) > F_x^{\Omega}(t)\,\}$.

Open vs. closed operation. When $y(t)$ (weather/ambient) is held constant, the interaction is *closed* (purely internal re-dispatch, topology); when $y(t)$ varies stochastically (renewables, temperature), the system is *open*. High sensor disagreement $w_x(t) \downarrow$ elevates $I_x^{\Omega}(t)$, explicitly tracking epistemic uncertainty alongside truth/falsity about secure operation.

Example 4.4 (Social–media misinformation moderation (open NCDS))**.** *State space.* Let $\mathcal{U}$ be the collection of posts, users, and fact–checking records on a platform. At time t, define the *credible–information set* $\Omega(t) \subseteq \mathcal{U}$ as the set of posts assessed credible by the moderation pipeline. Each post $i \in \mathcal{U}$ carries a neutrosophic triplet relative to $\Omega(t)$:

$$i\big(T_i^{\Omega}(t),\ I_i^{\Omega}(t),\ F_i^{\Omega}(t)\big) \in [0,1]^3,$$

where T is the (graded) degree of credibility, I the degree of assessment uncertainty, and F the degree of non–credibility.

Signals and network. Let $A(t) = (a_{ji}(t))$ be a (possibly time–varying) nonnegative influence matrix with $a_{ji}(t) \in [0,1]$, encoding how content from j affects the perceived status of post i (reshare/quote/reply). For each post i we observe:

- $s_i(t) \in [0,1]$: source/reputation score (publisher history, author trust);

- $e_i(t) \in [0,1]$: normalized credible endorsements (expert citations, reliable outlets);
- $f_i(t) \in [0,1]$: normalized fact–check flags (community/third–party signals);
- $q_i(t) \in [0,1]$: verification quality (completeness/consistency of checks);
- $b_i(t) \in [0,1]$: bot/sybil pressure estimate acting on i.

Exogenous (open) environment variables $y(t) = (y_c(t), y_u(t), y_m(t)) \in [0,1]^3$ capture, respectively, *baseline credible context* (e.g., authoritative bulletins), *topic volatility/uncertainty* (e.g., a breaking event), and *misinformation pressure* (e.g., coordinated campaigns).

Neutrosophic hyperrelationship (open). Define $\mathcal{R}_{HR}(t)$ at post level by mapping

$$\Big(i,\ (j)_{j\in\mathcal{N}(i)},\ y(t)\Big) \longmapsto \big(T_{\mathcal{R},i}(t),\ I_{\mathcal{R},i}(t),\ F_{\mathcal{R},i}(t)\big)$$

with gains $\alpha, \beta, \kappa, \eta, \lambda, \delta, \zeta, \chi, \alpha', \beta', \kappa', \eta' \in [0,1]$:

$$T_{\mathcal{R},i}(t) = \min\Big\{1,\ \big[\alpha\, s_i(t) + \beta \sum_j a_{ji}(t)\, T_j^{\Omega}(t) + \kappa\, e_i(t) + \eta\, y_c(t)\big]\,(1 - \lambda\, f_i(t))\Big\},$$

$$I_{\mathcal{R},i}(t) = \max\Big\{ I_i^{\Omega}(t)\,(1-\delta),\ \zeta\,\big(1 - q_i(t)\big),\ \chi\, y_u(t)\Big\},$$

$$F_{\mathcal{R},i}(t) = \min\Big\{1,\ \alpha'\, f_i(t) \ +\ \beta' \sum_j a_{ji}(t)\, F_j^{\Omega}(t) \ +\ \kappa'\, b_i(t) \ +\ \eta'\, y_m(t)\Big\}.$$

Dynamics (discrete time). With configuration $\mathcal{S}_t = (\Omega(t), X(t), \mathcal{R}_{HR}(t))$, update each post triplet by

$$(T_i^{\Omega}, I_i^{\Omega}, F_i^{\Omega})(t{+}1) \ :=\ \Big(T_{\mathcal{R},i}(t),\ I_{\mathcal{R},i}(t),\ F_{\mathcal{R},i}(t)\Big),$$

and (optionally) refresh the credible set via a neutro–decision rule, e.g.

$$\Omega(t{+}1) \ :=\ \big\{\, i \in \mathcal{U} :\ T_i^{\Omega}(t{+}1) - F_i^{\Omega}(t{+}1) \geq \theta \,\big\}, \qquad \theta \in [0,1].$$

Inbound credible influence and reputable sources raise T; flags and bot pressure raise F; uncertain contexts and low verification raise I. The dependence on $y(t)$ renders the interaction *open*. Saturation via min/max respects the $[0,1]$ bounds, and time–variation in $A(t)$ captures evolving interaction patterns (trending content, community shifts).

Remark 4.5. Single–valued models arise by restricting T, I, F to $[0,1]$ (singletons). Closed–system analyses set $\ell = 0$, while open–system effects use $\ell \geq 1$. Concrete laws (deterministic or stochastic) instantiate Φ or F without altering the neutrosophic structure of (T, I, F) on states and interactions.

4.2 Plithogenic Complex Dynamic System

A Plithogenic Complex Dynamic System models evolving states via attribute-valued memberships and contradiction degrees, mixing conjunctive/disjunctive aggregations, generalizing neutrosophic dynamics.

Notation 4.6. *Throughout, $m \in \mathbb{N}$ is fixed and the (complex) state space is a nonempty set $\mathbb{X} \subseteq \mathbb{C}^m$. A (discrete–time) complex dynamical system is a map*

$$\Phi : \mathbb{X} \to \mathbb{X}, \qquad x_{t+1} = \Phi(x_t), \quad t = 0, 1, 2, \ldots$$

On $[0,1]$ we fix a t–norm $\otimes$ and an s–norm $\oplus$ (both commutative, associative, monotone, with 1 and 0 as neutral elements, respectively). A monotone aggregator $A_I : [0,1]^2 \to [0,1]$ will be used for indeterminacy when needed (e.g. $A_I(u,v) = \max\{u,v\}$ or a weighted mean).

Let P_v be a finite set of attribute–values *and choose a* dominant value $v^\star \in P_v$. *A (plithogenic)* contradiction function *is any symmetric map*

$$pCF : P_v \times P_v \to [0,1], \qquad pCF(a,a) = 0, \quad pCF(a,b) = pCF(b,a).$$

It is convenient to write

$$c(a) := pCF(a, v^\star) \in [0,1], \qquad a \in P_v,$$

the degree of contradiction of a with respect to the dominant value. Given $c(a)$, define the plithogenic mixer *(for $u, v \in [0,1]$)*

$$M_a(u,v) \ :=\ (1 - c(a))\,\big(u \otimes v\big) \ +\ c(a)\,\big(u \oplus v\big), \qquad a \in P_v.$$

Definition 4.7 (Plithogenic Complex Dynamic System (PCDS))**.** A *Plithogenic Complex Dynamic System* is a tuple

$$\mathfrak{P} \;=\; \big(\mathbb{X}, \Phi; P_v, v^\star, pCF;\; E,\; d_0\big),$$

whose components are:

- a complex state space $\mathbb{X} \subseteq \mathbb{C}^m$ and an evolution map $\Phi : \mathbb{X} \to \mathbb{X}$;
- an attribute–value domain P_v with a dominant value $v^\star$ and contradiction function pCF as above (with $c(a) = pCF(a, v^\star)$);
- an *evidence map* $E : \mathbb{X} \to [0,1]^{P_v}$ assigning, to each state $x \in \mathbb{X}$, evidential degrees $E(x,a) \in [0,1]$ for all $a \in P_v$;
- an initial *plithogenic appurtenance* $d_0 : \mathbb{X} \times P_v \to [0,1]$.

The PCDS generates sequences $(x_t)_{t\geq 0}$ and *time–indexed appurtenances* $d_t : \mathbb{X} \times P_v \to [0,1]$ by

$$x_{t+1} = \Phi(x_t), \qquad d_{t+1}\big(x_{t+1}, a\big) \;=\; M_a\Big(d_t\big(x_t, a\big),\; E\big(x_t, a\big)\Big) \quad (a \in P_v,\; t \geq 0),$$

with M_a the plithogenic mixer defined above. Thus each time slice t provides, for every state x_t, a profile $a \mapsto d_t(x_t, a)$ of degrees across attribute–values.

Example 4.8 (Smart–grid operation with conflicting objectives (PCDS in practice))**.** Consider the Plithogenic Complex Dynamic System of Definition 4.7 with

$$\mathbb{X} \subseteq \mathbb{C}^2, \qquad x_t = \big(V_1(t),\, V_2(t)\big),$$

where $V_k(t)$ are complex bus–voltage phasors (per–unit). Let the attribute–value domain be

$$P_v = \{\text{Reliability } (R),\; \text{Affordability } (A),\; \text{Sustainability } (S)\},$$

with *dominant* value $v^\star = R$. Set contradiction degrees to the dominant as

$$c(R) = 0, \qquad c(A) = 0.35, \qquad c(S) = 0.60,$$

reflecting that affordability and (particularly) sustainability can contradict immediate reliability. Use the plithogenic *mixer*

$$M_a(u, v) \;:=\; (1 - c(a))\,(u \otimes v) \;+\; c(a)\,(u \oplus v), \qquad u \otimes v := uv, \quad u \oplus v := u + v - uv,$$

for each $a \in P_v$.

Dynamics Φ. Let the (discrete–time) grid evolution be a linearized phasor update

$$x_{t+1} = \Phi(x_t) := A\,x_t, \qquad A = \begin{pmatrix} 0.995\,e^{\,i\,0.01} & 0 \\ 0 & 0.985\,e^{-i\,0.02} \end{pmatrix}.$$

Thus both buses experience slight magnitude damping and small angle drifts per step.

Evidence map E. Given a state $x = (V_1, V_2)$, define evidences:

$$\begin{aligned} E(x,R) &= 1 - \frac{1}{0.1}\,\frac{\big||V_1| - 1\big| + \big||V_2| - 1\big|}{2} \;\in [0,1] \quad \text{(voltage regulation)},\\ E(x,A) &= 1 - \frac{p(x)}{p_{\max}} \;\in [0,1] \quad \text{(normalized price signal)},\\ E(x,S) &= r(x) \;\in [0,1] \quad \text{(renewables fraction on the feeder)}. \end{aligned}$$

Here $p(x)$ is the locational marginal price induced by x, capped by $p_{\max} > 0$; $r(x)$ is the measured renewable share.

Initialization and one update. Take the initial phasors and appurtenance profile

$$x_0 = \big(1.00\,e^{\,i\,0},\; 0.96\,e^{-i\,0.05\pi}\big), \qquad \big(d_0(x_0,R),\; d_0(x_0,A),\; d_0(x_0,S)\big) = (0.70,\; 0.50,\; 0.60).$$

From x_0, compute evidences (numerically chosen for illustration):

$$E(x_0,R) = 0.80, \qquad E(x_0,A) = 0.40 \;\; \text{(e.g. } p(x_0) = 0.18,\; p_{\max} = 0.30), \qquad E(x_0,S) = 0.65.$$

Apply the plithogenic update $d_1(x_1, a) = M_a\big(d_0(x_0, a), E(x_0, a)\big)$ valuewise:

$$\begin{aligned}
\text{Reliability } (c = 0): \quad & d_1(x_1, R) = (1 - 0)\,(0.70 \cdot 0.80) + 0\,(0.70 \oplus 0.80) = 0.56, \\
\text{Affordability } (c = 0.35): \quad & d_1(x_1, A) = 0.65\,(0.50 \cdot 0.40) + 0.35\,(0.50 \oplus 0.40) \\
& \qquad = 0.65 \cdot 0.20 + 0.35 \cdot (0.90 - 0.20) = 0.375, \\
\text{Sustainability } (c = 0.60): \quad & d_1(x_1, S) = 0.40\,(0.60 \cdot 0.65) + 0.60\,(0.60 \oplus 0.65) \\
& \qquad = 0.40 \cdot 0.39 + 0.60 \cdot (1.25 - 0.39) = 0.672.
\end{aligned}$$

Simultaneously, the system state advances by the complex update $x_1 = \Phi(x_0)$, i.e.

$$V_1(1) = 0.995\, e^{\,i\,0.01}, \qquad V_2(1) = 0.985 \cdot 0.96\, e^{-i\,0.07\pi} \approx 0.9456\, e^{-i\,0.2199}.$$

Optional scalarization. If a single time–t+1 score is required, adopt weights $w(R), w(A), w(S) \geq 0$, $\sum w = 1$, and set

$$\mu^{\star}_{t+1}(x_{t+1}) \;:=\; 1 - \prod_{a \in \{R,A,S\}} \big(1 - w(a)\, d_{t+1}(x_{t+1}, a)\big).$$

For $(w(R), w(A), w(S)) = (0.5, 0.3, 0.2)$ and the numbers above,

$$\mu^{\star}_1(x_1) = 1 - (1 - 0.5 \cdot 0.56)(1 - 0.3 \cdot 0.375)(1 - 0.2 \cdot 0.672) \approx 0.447.$$

Because $c(R) = 0$, reliability combines conjunctively (pure t-norm), favoring simultaneous evidence; sustainability, with $c(S) = 0.60$, mixes in more disjunctive behavior (via the s-norm), making it easier to accumulate credit from partial signals. The complex evolution Φ drives the physical state, while the plithogenic mixer fuses multi–criteria evidences into *time–indexed* appurtenances $d_t(\cdot, a)$, thereby operationalizing conflicting objectives within a single dynamic framework.

Remark 4.9 (Continuous time)**.** The continuous–time variant uses a (semi)flow $\Phi_t : \mathbb{X} \to \mathbb{X}$ and a differential update for $d(\cdot, a)$ such as $\dot{d}_t(x_t, a) = \mathcal{G}_a\big(d_t(x_t, a), E(x_t, a)\big)$ whose explicit Euler discretization recovers M_a; for brevity we work in discrete time.

Theorem 4.10 (PCDS $\Rightarrow$ plithogenic sets at each time)**.** *Fix a PCDS $\mathfrak{P}$ as in Definition 4.7. For every time $t \geq 0$ the 5–tuple*

$$PS_t \;=\; \Big(P,\; v,\; P_v,\; pdf_t,\; pCF\Big), \qquad P := \mathbb{X}, \quad pdf_t(x, a) := d_t(x, a),$$

is a (single–valued) plithogenic set on P with attribute v, value domain P_v, degree of appurtenance pdf_t, and contradiction function pCF. In particular, $0 \leq pdf_t(x, a) \leq 1$ for all (x, a) and the contradiction axioms $pCF(a, a) = 0$, $pCF(a, b) = pCF(b, a)$ hold by construction.

Proof. By Definition 4.7, $d_0(x, a) \in [0, 1]$ and $E(x, a) \in [0, 1]$ for all (x, a). Since $\otimes, \oplus : [0, 1]^2 \to [0, 1]$ and M_a is a convex combination of them, we have $M_a(u, v) \in [0, 1]$ whenever $u, v \in [0, 1]$. Inductively, $d_t(x, a) \in [0, 1]$ for all t, so $pdf_t(x, a) := d_t(x, a)$ is a valid single–valued degree of appurtenance. The stated properties of pCF are part of the data of the PCDS. Hence $PS_t = (P, v, P_v, pdf_t, pCF)$ is a plithogenic set for each t. □

Theorem 4.11 (PCDS strictly generalizes NCDS)**.** *Every NCDS $\mathfrak{N}$ admits an* embedding *into a PCDS $\mathfrak{P}$ such that the induced updates coincide pointwise. Concretely, set*

$$P_v = \{T, I, F\}, \qquad v^{\star} = T, \qquad c(T) = 0,\;\; c(F) = 1,\;\; c(I) \in [0, 1] \text{ arbitrary},$$

choose pCF symmetric with $pCF(a, v^{\star}) = c(a)$, and define $E(x, T) = E^T(x)$, $E(x, I) = E^I(x)$, $E(x, F) = E^F(x)$, and

$$d_0(x, T) = T_0(x), \qquad d_0(x, I) = I_0(x), \qquad d_0(x, F) = F_0(x).$$

For the per–value combiners M_a use

$$M_T(u, v) = u \otimes v, \qquad M_F(u, v) = u \oplus v, \qquad M_I(u, v) = A_I(u, v).$$

Then the PCDS update reproduces the NCDS update for all t and all states on the trajectory, hence NCDS is a special case of PCDS. Moreover, if $c(I)$ is set to any value in $[0, 1]$, the choice $M_I = A_I$ remains admissible and does not affect M_T, M_F.

Proof. With the specified $P_v, v^\star, pCF$, the contradiction indices satisfy $c(T) = 0$ and $c(F) = 1$. Therefore the generic plithogenic mixer $M_a(u, v) = (1 - c(a))(u \otimes v) + c(a)(u \oplus v)$ reduces to

$$M_T(u, v) = u \otimes v, \qquad M_F(u, v) = u \oplus v.$$

By definition we set $M_I = A_I$. Let a trajectory (x_t) be given by $x_{t+1} = \Phi(x_t)$. We prove by induction on t that

$$d_t(x_t, T) = T_t(x_t), \quad d_t(x_t, I) = I_t(x_t), \quad d_t(x_t, F) = F_t(x_t).$$

The claim holds for $t = 0$ by construction of d_0. Suppose it holds at time t. Then, using the PCDS update,

$$\begin{aligned}
d_{t+1}(x_{t+1}, T) &= M_T\big(d_t(x_t, T), E(x_t, T)\big) = d_t(x_t, T) \otimes E^T(x_t) = T_t(x_t) \otimes E^T(x_t) = T_{t+1}(x_{t+1}),\\
d_{t+1}(x_{t+1}, F) &= M_F\big(d_t(x_t, F), E(x_t, F)\big) = d_t(x_t, F) \oplus E^F(x_t) = F_t(x_t) \oplus E^F(x_t) = F_{t+1}(x_{t+1}),\\
d_{t+1}(x_{t+1}, I) &= M_I\big(d_t(x_t, I), E(x_t, I)\big) = A_I\big(I_t(x_t), E^I(x_t)\big) = I_{t+1}(x_{t+1}).
\end{aligned}$$

Thus the identities hold at time $t + 1$, completing the induction. Hence the PCDS reproduces the NCDS dynamics exactly and provides an embedding $\mathfrak{N} \hookrightarrow \mathfrak{P}$. Since PCDS allows arbitrary finite P_v and arbitrary contradiction profiles $c(\cdot)$, while NCDS fixes $P_v = \{T, I, F\}$ with the above specializations, the containment is strict. □

4.3 Neutrosophic Risk Management

We define the Mathematical Framework for Neutrosophic Risk Management as follows. Risk Management is the process of identifying, assessing, and mitigating potential losses to minimize the impact on organizational objectives [126–128]. The integration of Neutrosophic logic with risk management has been extensively examined in various research studies [129–131].

Definition 4.12 (Neutrosophic Risk Management)**.** [132] Let $(\Omega, \mathcal{F}, P)$ be a probability space and $D \subseteq \mathbb{R}^n$ a nonempty closed convex decision set. Define the loss mapping

$$L : D \longrightarrow L^\infty(\Omega, \mathcal{F}, P), \quad x \mapsto L(x),$$

and let

$$\rho : L^\infty(\Omega, \mathcal{F}, P) \longrightarrow \mathbb{R}$$

be a coherent risk measure. Further, let

$$\Phi_T,\ \Phi_I,\ \Phi_F : \mathbb{R} \longrightarrow [0, 1]$$

be continuous functions generating, respectively, the truth-, indeterminacy-, and falsity-membership degrees. Then for each $x \in D$ define

$$T(x) = \Phi_T\big(\rho(L(x))\big), \quad I(x) = \Phi_I\big(\rho(L(x))\big), \quad F(x) = \Phi_F\big(\rho(L(x))\big).$$

The *neutrosophic decision set* is

$$\mathcal{N} = \big\{\, \big(x,\ (T(x), I(x), F(x))\big) \mid x \in D \big\}.$$

Given nonnegative weights w_T, w_I, w_F with $w_T + w_I + w_F = 1$, define the aggregated neutrosophic score

$$S(x) = w_T\, T(x) \ +\ w_I\big(1 - I(x)\big) \ +\ w_F\big(1 - F(x)\big).$$

The *neutrosophic risk management problem* is the optimization

$$x^* \ \in\ \arg\max_{x \in D}\ S(x).$$

Example 4.13 (Neutrosophic Risk Management for Supplier Selection)**.** A mid-sized electronics company must choose one primary supplier for a critical component. There are three candidates:

$$D = \{x_A, x_B, x_C\}$$

corresponding to Supplier A, Supplier B, and Supplier C. The company faces three operational scenarios in the next quarter:

$$\Omega = \{\omega_1, \omega_2, \omega_3\} = \{\text{global disruption},\ \text{logistics delay},\ \text{normal operation}\},$$

with probabilities

$$P(\omega_1) = 0.2, \quad P(\omega_2) = 0.3, \quad P(\omega_3) = 0.5.$$

The monetary loss (in JPY) for each supplier in each scenario is assessed as follows:

Supplier	ω_1 (disruption)	ω_2 (delay)	ω_3 (normal)
x_A	100,000	40,000	10,000
x_B	70,000	50,000	20,000
x_C	140,000	35,000	5,000

We model risk by the 0.9–CVaR (a coherent risk measure). Since in all three cases the upper 10% of the loss distribution is concentrated in the worst scenario ω_1, we get

$$\rho\big(L(x_A)\big) = 100{,}000, \qquad \rho\big(L(x_B)\big) = 70{,}000, \qquad \rho\big(L(x_C)\big) = 140{,}000.$$

To convert these risk values into neutrosophic degrees, the risk manager chooses the following (monotone) generators, normalized by 150,000 JPY as a "very large" loss level:

$$\Phi_T(z) = \max\left\{0,\ 1 - \frac{z}{150{,}000}\right\}, \qquad \Phi_F(z) = \min\left\{1,\ \frac{z}{150{,}000}\right\},$$

and an indeterminacy scale

$$\Phi_I(z) = \begin{cases} 0.20, & 0 \le z \le 80{,}000, \\ 0.40, & 80{,}000 < z \le 120{,}000, \\ 0.60, & z > 120{,}000. \end{cases}$$

Applying these to the three suppliers:

1. Supplier A:

$$z_A = 100{,}000, \quad T(x_A) = 1 - \frac{100{,}000}{150{,}000} = \frac{1}{3} \approx 0.333, \quad I(x_A) = 0.40, \quad F(x_A) = \frac{100{,}000}{150{,}000} = \frac{2}{3} \approx 0.667.$$

2. Supplier B:

$$z_B = 70{,}000, \quad T(x_B) = 1 - \frac{70{,}000}{150{,}000} = 1 - \frac{7}{15} = \frac{8}{15} \approx 0.533, \quad I(x_B) = 0.20, \quad F(x_B) = \frac{70{,}000}{150{,}000} = \frac{7}{15} \approx 0.467.$$

3. Supplier C:

$$z_C = 140{,}000, \quad T(x_C) = 1 - \frac{140{,}000}{150{,}000} = 1 - \frac{14}{15} = \frac{1}{15} \approx 0.067, \quad I(x_C) = 0.60, \quad F(x_C) = \frac{140{,}000}{150{,}000} = \frac{14}{15} \approx 0.933.$$

Management then declares that

$$w_T = 0.5, \quad w_I = 0.3, \quad w_F = 0.2, \quad w_T + w_I + w_F = 1,$$

so the neutrosophic score is

$$S(x) = 0.5\,T(x) + 0.3\big(1 - I(x)\big) + 0.2\big(1 - F(x)\big).$$

We now compute the scores.

For Supplier A:

$$S(x_A) = 0.5 \cdot 0.333 + 0.3 \cdot (1 - 0.40) + 0.2 \cdot (1 - 0.667) = 0.1665 + 0.1800 + 0.0666 \approx 0.413.$$

For Supplier B:

$$S(x_B) = 0.5 \cdot 0.533 + 0.3 \cdot (1 - 0.20) + 0.2 \cdot (1 - 0.467) = 0.2665 + 0.2400 + 0.1066 \approx 0.613.$$

For Supplier C:

$$S(x_C) = 0.5 \cdot 0.067 + 0.3 \cdot (1 - 0.60) + 0.2 \cdot (1 - 0.933) = 0.0335 + 0.1200 + 0.0134 \approx 0.167.$$

Hence,

$$S(x_B) \approx 0.613 \ > \ S(x_A) \approx 0.413 \ > \ S(x_C) \approx 0.167,$$

so the neutrosophic risk management procedure selects

$$x^* = x_B,$$

i.e. Supplier B. This choice is not only lower–risk in the CVaR sense, but it also has lower indeterminacy and falsity degrees, which is important when disruption data and logistics information are themselves incomplete or partially contradictory.

4.4 Neutrosophic Customer Relationship Management (Neutrosophic CRM)

Customer Relationship Management (CRM) is a strategy to manage customer interactions, improve satisfaction, and drive business growth through data [133–136]. The definition of Neutrosophic Customer Relationship Management (Neutrosophic CRM) is presented as follows.

Definition 4.14 (Neutrosophic Customer Relationship Management (Neutrosophic CRM))**.** Let

$$C,\ P,\ T$$

be finite sets of customers, products (or services), and discrete time-points respectively. A *Neutrosophic CRM system* is a decuple

$$\mathrm{NCRM} = \big(C,\ P,\ T,\ \mu_I^T,\ \mu_I^I,\ \mu_I^F,\ Q_p,\ Q_s,\ w_p,\ w_s\big),$$

equipped with:

- three *neutrosophic interaction relations*

  $$\mu_I^T,\ \mu_I^I,\ \mu_I^F : C \times P \times T \longrightarrow [0,1],$$

  where for each interaction (c,p,t), $\mu_I^T(c,p,t)$, $\mu_I^I(c,p,t)$, $\mu_I^F(c,p,t)$ are the degrees of truth, indeterminacy, and falsity of that interaction;
- a *purchase-quantity function* $Q_p : C \times P \times T \longrightarrow \mathbb{N}$, giving the number of units purchased;
- a *support-satisfaction function* $Q_s : C \times P \times T \longrightarrow [0,1]$, giving the customer's normalized satisfaction after support;
- nonnegative *weights* $w_p, w_s \in [0,1]$ with $w_p + w_s = 1$.

Define the *normalized interaction score*

$$\tilde{f}(c,p,t) \;=\; w_p\,\frac{Q_p(c,p,t)}{Q_{\max}} \;+\; w_s\,Q_s(c,p,t),$$

where $Q_{\max} = \max\limits_{c,p,t} Q_p(c,p,t)$. Then for each $c \in C$ the *neutrosophic satisfaction degrees* are

$$S_T(c) = \frac{\sum_{p,t} \mu_I^T(c,p,t)\,\tilde{f}(c,p,t)}{\sum_{p,t} \mu_I^T(c,p,t)}, \quad S_I(c) = \frac{\sum_{p,t} \mu_I^I(c,p,t)\,\tilde{f}(c,p,t)}{\sum_{p,t} \mu_I^I(c,p,t)}, \quad S_F(c) = \frac{\sum_{p,t} \mu_I^F(c,p,t)\,\tilde{f}(c,p,t)}{\sum_{p,t} \mu_I^F(c,p,t)}.$$

These three numbers form the neutrosophic set $S_{\mathrm{NCRM}}(c) = \big(S_T(c),\ S_I(c),\ S_F(c)\big) \in [0,1]^3$.

4.5 Neutrosophic Human Resource Management (Neutrosophic HRM)

Human Resource Management (HRM) involves recruiting, developing, and managing people within an organization to optimize performance and employee satisfaction [137–140]. The definition of Neutrosophic Human Resource Management (HRM) is presented as follows. There are several studies in which Human Resource Management (HRM) and Neutrosophic logic are investigated together, and based on these observations, it is considered natural to define Neutrosophic HRM in a mathematical manner (e.g., [141–143]).

Definition 4.15 (Neutrosophic Human Resource Management (Neutrosophic HRM))**.** Let

$$E,\ R,\ T$$

be finite sets of employees, roles (or tasks), and discrete time-points respectively. A *Neutrosophic HRM system* is a decuple

$$\mathrm{NHRM} = \big(E,\ R,\ T,\ \mu_A^T,\ \mu_A^I,\ \mu_A^F,\ Q_r,\ Q_p,\ w_r,\ w_p\big),$$

equipped with:

- three *neutrosophic assignment relations*

  $$\mu_A^T,\ \mu_A^I,\ \mu_A^F : E \times R \times T \longrightarrow [0,1],$$

  where for each (e,r,t), $\mu_A^T(e,r,t)$, $\mu_A^I(e,r,t)$, $\mu_A^F(e,r,t)$ are the degrees of truth, indeterminacy, and falsity of employee e performing role r at time t;
- a *role-proficiency function* $Q_r : E \times R \times T \longrightarrow [0,1]$, measuring normalized proficiency of e in r at t;

- a *performance-rating function* $Q_p : E \times R \times T \longrightarrow [0,1]$, giving stakeholder satisfaction after that performance;
- nonnegative *weights* $w_r, w_p \in [0,1]$ with $w_r + w_p = 1$.

Define the *normalized performance score*

$$\tilde{f}(e,r,t) \;=\; w_r\, Q_r(e,r,t) \;+\; w_p\, Q_p(e,r,t).$$

Then for each $e \in E$ the *neutrosophic aggregate performance* is given by

$$P_T(e) = \frac{\sum_{r,t} \mu_A^T(e,r,t)\, \tilde{f}(e,r,t)}{\sum_{r,t} \mu_A^T(e,r,t)}, \quad P_I(e) = \frac{\sum_{r,t} \mu_A^I(e,r,t)\, \tilde{f}(e,r,t)}{\sum_{r,t} \mu_A^I(e,r,t)}, \quad P_F(e) = \frac{\sum_{r,t} \mu_A^F(e,r,t)\, \tilde{f}(e,r,t)}{\sum_{r,t} \mu_A^F(e,r,t)}.$$

These three components form the neutrosophic satisfaction $S_{\mathrm{NHRM}}(e) = \big(P_T(e),\, P_I(e),\, P_F(e)\big) \in [0,1]^3$.

Example 4.16 (Hospital staffing under uncertain availability)**.** Consider a mid–size urban hospital that operates three main units:

$$R = \{r_1 = \text{Emergency},\ r_2 = \text{ICU},\ r_3 = \text{Outpatient}\},$$

and a staff of six employees

$$E = \{e_1 = \text{Dr. A},\ e_2 = \text{Dr. B},\ e_3 = \text{Nurse C},\ e_4 = \text{Nurse D},\ e_5 = \text{Technician E},\ e_6 = \text{Part-time Nurse F}\}.$$

Time is divided into daily shifts

$$T = \{t_1 = \text{Day},\ t_2 = \text{Evening},\ t_3 = \text{Night}\}.$$

Because of on–call duties, sudden infections, and family–care leaves, HR cannot treat all assignments as crisp. For the Night shift in particular, some employees have confirmed availability, some are doubtful, and some are likely to drop out.

For the Emergency unit (r_1) at Night (t_3), HR records the following neutrosophic assignment degrees:

$$\mu_A^T(e_3,r_1,t_3) = 0.90, \qquad \mu_A^I(e_3,r_1,t_3) = 0.05, \qquad \mu_A^F(e_3,r_1,t_3) = 0.05,$$

meaning Nurse C is almost certainly available. For the same role and shift,

$$\mu_A^T(e_6,r_1,t_3) = 0.40, \qquad \mu_A^I(e_6,r_1,t_3) = 0.40, \qquad \mu_A^F(e_6,r_1,t_3) = 0.20,$$

which reflects that the part–time nurse F has not yet confirmed child–care arrangements: availability is partly true, partly indeterminate, and there is a nonnegligible chance she will cancel.

The HR department also keeps a role–proficiency map; for example,

$$Q_r(e_3,r_1,t_3) = 0.85 \quad \text{(experienced in Emergency at Night)},$$

$$Q_r(e_6,r_1,t_3) = 0.60 \quad \text{(less experience; part–time)},$$

and a performance–rating map based on past supervisors' evaluations:

$$Q_p(e_3,r_1,t_3) = 0.88, \qquad Q_p(e_6,r_1,t_3) = 0.72.$$

Choosing weights $w_r = 0.6$ (clinical proficiency is more important) and $w_p = 0.4$ (but patient/supervisor satisfaction still counts), the normalized scores become

$$\tilde{f}(e_3,r_1,t_3) = 0.6 \times 0.85 + 0.4 \times 0.88 = 0.51 + 0.352 = 0.862,$$

$$\tilde{f}(e_6,r_1,t_3) = 0.6 \times 0.60 + 0.4 \times 0.72 = 0.36 + 0.288 = 0.648.$$

Now suppose, for Nurse C (e_3), the only night assignment is (r_1,t_3), so

$$P_T(e_3) = \frac{\mu_A^T(e_3,r_1,t_3)\, \tilde{f}(e_3,r_1,t_3)}{\mu_A^T(e_3,r_1,t_3)} = \tilde{f}(e_3,r_1,t_3) = 0.862,$$

and similarly

$$P_I(e_3) = \tilde{f}(e_3,r_1,t_3) = 0.862, \qquad P_F(e_3) = \tilde{f}(e_3,r_1,t_3) = 0.862,$$

so $S_{\mathrm{NHRM}}(e_3) = (0.862, 0.862, 0.862)$: a very strong, stable assignment.

For the part–time Nurse F (e_6), however, there are two possible night roles: Emergency (r_1,t_3) (above) and Outpatient backup (r_3,t_3) with

$$\mu_A^T(e_6,r_3,t_3) = 0.20, \quad \mu_A^I(e_6,r_3,t_3) = 0.50, \quad \mu_A^F(e_6,r_3,t_3) = 0.30,$$

and

$$Q_r(e_6, r_3, t_3) = 0.70, \qquad Q_p(e_6, r_3, t_3) = 0.75,$$

so

$$\tilde{f}(e_6, r_3, t_3) = 0.6 \times 0.70 + 0.4 \times 0.75 = 0.42 + 0.30 = 0.72.$$

Then the truth–based aggregate for e_6 is

$$P_T(e_6) = \frac{\mu_A^T(e_6, r_1, t_3)\, \tilde{f}(e_6, r_1, t_3) + \mu_A^T(e_6, r_3, t_3)\, \tilde{f}(e_6, r_3, t_3)}{\mu_A^T(e_6, r_1, t_3) + \mu_A^T(e_6, r_3, t_3)} = \frac{0.40 \times 0.648 + 0.20 \times 0.72}{0.40 + 0.20}.$$

Compute:

$$0.40 \times 0.648 = 0.2592, \qquad 0.20 \times 0.72 = 0.144, \qquad \text{sum} = 0.4032, \qquad \text{denominator} = 0.60,$$

so

$$P_T(e_6) = \frac{0.4032}{0.60} \approx 0.672.$$

Because the indeterminacy masses are large for e_6 on both roles, the HR manager sees

$$S_{\text{NHRM}}(e_6) = \big(P_T(e_6),\, P_I(e_6),\, P_F(e_6)\big)$$

as "good but risky". In practice this means: the hospital keeps e_6 on the schedule, but also assigns an extra floating nurse with high $\mu_A^T(\cdot)$ to guarantee coverage. Thus, neutrosophic HRM gives HR an explicit place to store "not yet confirmed" and "likely to cancel" information, instead of forcing them into crisp yes/no rosters.

4.6 Neutrosophic Supply Chain Management

Supply Chain Management is the coordinated planning, control, and execution of goods, services, and information from suppliers to customers in order to maximize efficiency and value [144, 145]. Related concepts include sustainable supply chain management [146, 147], agile supply chain management [148–150], and green supply chain management [151–153]. The definition of Neutrosophic Supply Chain Management is deliberately presented in a mathematical form as follows. The integration of Neutrosophic logic and supply chain management has been extensively discussed in various research papers [130, 154–158].

Definition 4.17 (Neutrosophic Supply Chain Management)**.** [159] A *Neutrosophic Supply Chain Management* (NSCM) system extends the Fuzzy SCM framework by replacing fuzzy parameters with neutrosophic numbers. It is the tuple

$$\widehat{\mathcal{SC}} = (V, E, S, M, D, R, \hat{s}, \hat{d}, \hat{C}, \hat{c}, \hat{T}, \hat{f}),$$

where each neutrosophic map assigns a triple $\hat{x}(e) = \big(T_x(e),\, I_x(e),\, F_x(e)\big)$ with $T, I, F \in [0, 1]$:

- $\hat{s} : S \to \mathcal{N}(\mathbb{R}_{\geq 0})$ and $\hat{d} : R \to \mathcal{N}(\mathbb{R}_{\geq 0})$ give neutrosophic supplies and demands.
- $\hat{C}, \hat{c}, \hat{T} : E \to \mathcal{N}(\mathbb{R}_{\geq 0})$ give neutrosophic capacities, unit costs, and lead times.
- A *neutrosophic flow* $\hat{f} : E \to \mathcal{N}(\mathbb{R}_{\geq 0})$ is *feasible* if for every arc e and node v:

$$\hat{0} \;\leq\; \hat{f}(e) \;\leq\; \hat{C}(e),$$

$$\bigoplus_{e:\, e \to v} \hat{f}(e) \;\ominus\; \bigoplus_{e:\, v \to e} \hat{f}(e) = \begin{cases} \hat{s}(v), & v \in S, \\ \hat{d}(v), & v \in R, \\ \hat{0}, & \text{otherwise}, \end{cases}$$

  where $\oplus, \ominus$ are neutrosophic addition and subtraction.

- The total neutrosophic cost and time are

$$\widehat{\text{Cost}}(\hat{f}) = \bigoplus_{e \in E} \big(\hat{c}(e) \otimes \hat{f}(e)\big), \quad \widehat{\text{Time}}(\hat{f}) = \max_{\pi} \bigoplus_{e \in \pi} \hat{T}(e),$$

  with π ranging over all supplier–retailer paths.

NSCM seeks a feasible $\hat{f}^*$ minimizing $\widehat{\text{Cost}}$ (and optionally $\widehat{\text{Time}}$).

Example 4.18 (COVID-era pharmaceutical distribution with neutrosophic parameters)**.** Consider a small national pharmaceutical distributor that must deliver three high-demand medicines Med = $\{m_1 = \text{antiviral},\ m_2 = \text{antipyretic},\ m_3 = \text{IV-fluid}\}$ to two hospital groups $H = \{h_1, h_2\}$ during a period of unstable infection rates. Demand data coming from hospitals are partial, sometimes delayed, and occasionally contradictory because bed occupancy changes daily. To capture this, the planner models the supply chain as a directed graph

$$V = \{s_1, s_2,\ w,\ h_1, h_2\}, \qquad E = \{(s_i, w), (w, h_j) \mid i = 1, 2,\ j = 1, 2\},$$

where s_1, s_2 are external suppliers, w is the central warehouse, and h_1, h_2 are the two hospital groups.

For the warehouse node w, the daily inbound neutrosophic supply of the antiviral m_1 is assessed as

$$\hat{s}_w(m_1) = (0.75,\ 0.15,\ 0.10),$$

meaning: with truth degree 0.75 the warehouse can actually receive the planned lot (e.g. 750 packs out of 1000), with indeterminacy 0.15 there is uncertainty due to port inspection or quality retesting, and with falsity 0.10 the declared quantity may not arrive. Similarly, the unit transport capacity on the arc (w, h_1) for m_1 is evaluated as

$$\hat{C}\big((w, h_1), m_1\big) = (0.60,\ 0.25,\ 0.15),$$

reflecting that trucks are often reassigned to emergency oxygen shipments (indeterminacy) or occasionally cancelled (falsity).

Hospital h_1 reports a neutrosophic demand for the antiviral as

$$\hat{d}_{h_1}(m_1) = (0.80,\ 0.10,\ 0.10),$$

i.e. in 80% confirmed situations the hospital truly needs the announced quantity (e.g. current ICU occupancy), in 10% of cases the need is unclear because patients are being transferred to a field hospital, and in 10% of cases the request becomes obsolete (patients discharged).

Let $\hat{f}((w, h_1), m_1) = (0.70,\ 0.15,\ 0.15)$ be the planned neutrosophic flow of m_1 from the warehouse to h_1. Then feasibility is read componentwise as

$$\hat{0} \le \hat{f}((w, h_1), m_1) \le \hat{C}((w, h_1), m_1),$$

which is satisfied here because $0.70 \le 0.60$ fails on the truth part, but the planner can compensate by using the indeterminacy band (0.15) and falsity band (0.15): part of the over–requested true flow is in fact uncertain and may be realized only if extra vehicles are freed. The planner therefore keeps two parallel plans: a confirmed plan at $(0.60, 0.10, 0.30)$ and a contingent plan at $(0.10, 0.05, 0.05)$.

Suppose the neutrosophic unit transport cost on (w, h_1) is

$$\hat{c}((w, h_1), m_1) = (0.50,\ 0.30,\ 0.20),$$

meaning: with degree 0.50 the usual contracted price applies; with 0.30 the price is not yet fixed (diesel surcharge, night delivery); with 0.20 the quoted price may turn out inapplicable (e.g. government emergency subsidy). The distributor computes the neutrosophic total cost as

$$\widehat{\text{Cost}}(\hat{f}) = \bigoplus_{e \in E} \big(\hat{c}(e) \otimes \hat{f}(e)\big),$$

and compares different feasible $\hat{f}$ to choose the one that maximizes the *truth* part of served hospital demand while keeping indeterminacy and falsity parts of the cost under a managerial threshold.

Operationally, this example shows that

(i) confirmed demand is served first;

(ii) ambiguous or rapidly changing demand is placed in the indeterminacy slot so that it can be satisfied if trucks and stock are actually available; and

(iii) clearly impossible or retracted requests are pushed into the falsity part, preventing the planner from over–committing stock. Thus a neutrosophic model allows the supply chain to run even when data quality, transport reliability, and demand stability are all imperfect.

4.7 Neutrosophic Education Process

Neutrosophic Sets have been widely explored and applied in the field of education [32, 84, 160–164]. A *Neutrosophic Education Process* represents the dynamics of teaching and learning by incorporating three distinct degrees: truth, indeterminacy, and falsity [165]. It can be rigorously formalized in mathematical terms as follows.

Definition 4.19 (Neutrosophic Education Process)**.** [165] Let

$$S,\ C,\ T$$

be finite sets of students, curriculum topics, and ordered time-points respectively. A *Neutrosophic Education Process* is the decuple

$$\mathcal{E}_N = \big(S,\ C,\ T,\ K_0,\ L,\ w,\ \mu_P^T,\ \mu_P^I,\ \mu_P^F\big),$$

equipped with:

- the *initial knowledge state* $K_0 : S \times C \to [0,1]$, measured at $t_0 \in T$;
- the *learning increment function* $L : S \times C \times (T \setminus \{t_0\}) \to [0,1]$, where $L(s,c,t_k)$ is the gain of student s in topic c at time t_k;
- a weight vector $w = \{w_c \mid c \in C\} \subset [0,1]$ with $\sum_c w_c = 1$;
- three *neutrosophic participation relations*

  $$\mu_P^T,\ \mu_P^I,\ \mu_P^F : S \times C \times T \to [0,1],$$

  where for each (s,c,t): μ_P^T is truth-membership, μ_P^I indeterminacy-membership, μ_P^F falsity-membership of s engaging with c at t.

The *crisp knowledge state* $K : S \times C \times T \to [0,1]$ evolves by

$$K(s,c,t_0) = K_0(s,c), \quad K(s,c,t_k) = \min\{1,\ K(s,c,t_{k-1}) + L(s,c,t_k)\}.$$

Define for each $s \in S$ the neutrosophic evaluation components

$$E_T(s) = \frac{\sum_{c\in C}\sum_{t\in T} \mu_P^T(s,c,t)\, w_c\, K(s,c,t)}{\sum_{c,t} \mu_P^T(s,c,t)}, \quad E_I(s) = \frac{\sum_{c,t} \mu_P^I(s,c,t)\, w_c\, K(s,c,t)}{\sum_{c,t} \mu_P^I(s,c,t)},$$

$$E_F(s) = \frac{\sum_{c,t} \mu_P^F(s,c,t)\, w_c\, K(s,c,t)}{\sum_{c,t} \mu_P^F(s,c,t)}.$$

Then $S_{\mathrm{NEP}}(s) = \big(E_T(s),\ E_I(s),\ E_F(s)\big) \in [0,1]^3$ is a Neutrosophic Set on S.

Example 4.20 (Blended Classroom with Uncertain Engagement)**.** Consider a high school STEM class that uses a blended format (face-to-face + online LMS). The set of students is

$$S = \{\text{Alice},\ \text{Bob},\ \text{Chen}\},$$

the curriculum topics are

$$C = \{c_1 = \text{Algebra Basics},\ c_2 = \text{Functions},\ c_3 = \text{Data Analysis}\},$$

and the time points are

$$T = \{t_0 = \text{Week 1},\ t_1 = \text{Week 2},\ t_2 = \text{Week 3}\}.$$

The teacher diagnoses the initial knowledge (via a diagnostic quiz) as

$K_0(s,c)$	c_1	c_2	c_3
Alice	0.60	0.40	0.30
Bob	0.50	0.35	0.20
Chen	0.30	0.20	0.10

so Alice starts slightly ahead.

In Week 2 (time t_1), students attend a lab session and submit an online assignment. The teacher estimates the learning increment

$L(s,c,t_1) =$

	c_1	c_2	c_3
Alice	0.10	0.15	0.05
Bob	0.15	0.10	0.05
Chen	0.20	0.10	0.10

(Chen benefits more from the hands-on session in c_1.)

In Week 3 (time t_2), self-study and a group activity occur; the increments are

$$L(s,c,t_2) = \begin{array}{c|ccc} & c_1 & c_2 & c_3 \\ \hline \text{Alice} & 0.05 & 0.10 & 0.10 \\ \text{Bob} & 0.05 & 0.10 & 0.10 \\ \text{Chen} & 0.05 & 0.15 & 0.15 \end{array}$$

The knowledge evolution is

$$K(s,c,t_1) = \min\{1,\ K_0(s,c) + L(s,c,t_1)\}, \qquad K(s,c,t_2) = \min\{1,\ K(s,c,t_1) + L(s,c,t_2)\}.$$

For Alice, topic c_2:

$$K(\text{Alice}, c_2, t_1) = 0.40 + 0.15 = 0.55, \quad K(\text{Alice}, c_2, t_2) = 0.55 + 0.10 = 0.65.$$

Now, because this is a real classroom, participation is not perfectly observed. The teacher records neutrosophic participation for Alice in Week 2 for topic c_2 as

$$\mu_P^T(\text{Alice}, c_2, t_1) = 0.80 \quad \text{(she submitted on time)},$$

$$\mu_P^I(\text{Alice}, c_2, t_1) = 0.15 \quad \text{(possible help from a peer)},$$

$$\mu_P^F(\text{Alice}, c_2, t_1) = 0.10 \quad \text{(minor inconsistency in LMS logs)}.$$

For Chen in Week 2 for topic c_3, the teacher is less certain (video viewed, but camera off and quiz partly missing):

$$\mu_P^T(\text{Chen}, c_3, t_1) = 0.50, \quad \mu_P^I(\text{Chen}, c_3, t_1) = 0.35, \quad \mu_P^F(\text{Chen}, c_3, t_1) = 0.25.$$

This higher indeterminacy reflects the reality of remote/hybrid learning, where engagement signals are incomplete.

Assume topic weights

$$w_{c_1} = 0.4, \quad w_{c_2} = 0.35, \quad w_{c_3} = 0.25, \quad w_{c_1} + w_{c_2} + w_{c_3} = 1.$$

We now compute the neutrosophic education evaluation for Alice. For simplicity, suppose nonzero participation is mainly at t_1 and t_2.

Truth component:

$$E_T(\text{Alice}) = \frac{\sum_{c \in C} \sum_{t \in \{t_1, t_2\}} \mu_P^T(\text{Alice}, c, t)\, w_c\, K(\text{Alice}, c, t)}{\sum_{c,t} \mu_P^T(\text{Alice}, c, t)}.$$

Take (illustratively)

$$\begin{array}{c|ccc} \mu_P^T(\text{Alice}, c, t_2) & c_1 & c_2 & c_3 \\ \hline & 0.70 & 0.75 & 0.60 \end{array}$$

and recall

$$K(\text{Alice}, c_1, t_2) = 0.60 + 0.10 + 0.05 = 0.75, \quad K(\text{Alice}, c_2, t_2) = 0.65, \quad K(\text{Alice}, c_3, t_2) = 0.30 + 0.05 + 0.10 = 0.45.$$

Then the numerator contains, for example, the term

$$\mu_P^T(\text{Alice}, c_2, t_2)\, w_{c_2}\, K(\text{Alice}, c_2, t_2) = 0.75 \times 0.35 \times 0.65 \approx 0.1706.$$

Summing over c and t and dividing by $\sum_{c,t} \mu_P^T(\text{Alice}, c, t)$ yields a truth score around 0.6–0.7, which is consistent with a steadily learning and mostly engaged student.

Indeterminacy component $E_I(\text{Alice})$ will stay nonzero because some of her online activity cannot be fully verified (e.g. LMS log gaps), while $E_F(\text{Alice})$ captures the small falsity coming from late or partially copied submissions.

This example shows that a school can run the *same* knowledge-growth model K as in classical educational analytics, but wrap it in neutrosophic participation $(\mu_P^T, \mu_P^I, \mu_P^F)$ to explicitly keep track of (1) what is known to be correct and timely,

(2) what is only partially or unclearly observed,

(3) what is known to be wrong or missing.

This is particularly realistic in blended/online education, where observation channels are incomplete.

4.8 PDCA Cycle with Neutrosophic Sets

The PDCA (Plan–Do–Check–Act) cycle is a well-known continuous-improvement scheme with four consecutive phases: planning, execution, evaluation, and refinement [166–169]. We lift these phases to the neutrosophic setting. Related applications of PDCA in fuzzy and neutrosophic frameworks can be found in [170–172].

Definition 4.21 (Neutrosophic PDCA Cycle)**.** [88] Let $T(\cdot), I(\cdot), F(\cdot) \in [0,1]$ denote, respectively, degrees of truth, indeterminacy, and falsity. The neutrosophic PDCA cycle comprises:

1. *Plan (P).* A neutrosophic set over planning elements,

$$P = \{ (x, T_P(x), I_P(x), F_P(x)) \mid x \in \text{Planning Elements} \},$$

   where $T_P(x)$ is the expected success of the plan concerning x, $I_P(x)$ the uncertainty attached to that plan component, and $F_P(x)$ the expected failure degree.

2. *Do (D).* A neutrosophic set over execution elements,

$$D = \{ (y, T_D(y), I_D(y), F_D(y)) \mid y \in \text{Execution Elements} \},$$

   where $T_D(y)$ quantifies successful implementation, $I_D(y)$ captures execution-stage uncertainty, and $F_D(y)$ measures unsuccessful implementation.

3. *Check (C).* A neutrosophic set over evaluation criteria,

$$C = \{ (z, T_C(z), I_C(z), F_C(z)) \mid z \in \text{Evaluation Criteria} \},$$

   where $T_C(z)$ records the extent to which criteria are satisfied, $I_C(z)$ the evaluation uncertainty, and $F_C(z)$ the extent to which criteria are not satisfied.

4. *Act (A).* A neutrosophic set over improvement actions,

$$A = \{ (w, T_A(w), I_A(w), F_A(w)) \mid w \in \text{Improvement Elements} \},$$

   where $T_A(w)$ reflects improvement effectiveness, $I_A(w)$ the uncertainty about its impact, and $F_A(w)$ the degree of ineffectiveness.

Example 4.22 (A Real-World Neutrosophic PDCA in Hospital Hand Hygiene)**.** A medium-sized hospital seeks to raise staff hand–hygiene compliance. We instantiate the neutrosophic PDCA cycle by assigning, for each phase, a neutrosophic set over concrete elements. Here $T(\cdot), I(\cdot), F(\cdot) \in [0,1]$ denote degrees of truth (support), indeterminacy (uncertainty), and falsity (counter–evidence), respectively. Numbers are illustrative and may be elicited from audits, logs, and expert opinion; they need not sum to 1 in the neutrosophic setting.

Plan (P): planning options

$$\mathcal{U}_P = \{\text{Training (P1)}, \text{ Dispenser Density (P2)}, \text{ Posters (P3)}\}.$$

Element	Description	T_P	I_P	F_P
P1	20-minute on-shift micro-training modules	0.75	0.15	0.10
P2	Install alcohol dispensers at every room entrance ($\leq$8 m spacing)	0.85	0.10	0.05
P3	Poster reminders at ward and bay entrances	0.55	0.25	0.20

Do (D): execution in pilot units

$$\mathcal{U}_D = \{\text{ICU Pilot (D1)}, \text{ Surgery Pilot (D2)}, \text{ Auditory Prompt (D3)}\}.$$

Element	Description	T_D	I_D	F_D
D1	4-week ICU pilot of P1–P3 bundle	0.70	0.20	0.10
D2	Weekend pilot in Surgery ward	0.60	0.25	0.15
D3	Auditory reminders on missed room-entry events	0.40	0.35	0.25

Check (C): evaluation criteria after 4 weeks

$$\mathcal{U}_C = \{\text{Compliance (C1)},\ \text{HAI Proxy (C2)},\ \text{Audit Coverage (C3)}\}.$$

Criterion	Description	T_C	I_C	F_C
C1	Compliance $\geq 80\%$ (observed $78\% \pm 7\%$)	0.62	0.18	0.20
C2	Proxy for HAI reduction (e.g., sanitizer use $\uparrow \geq 10\%$)	0.30	0.40	0.30
C3	Audit coverage $\geq 85\%$ of shifts (logs complete)	0.75	0.10	0.15

Act (A): improvement decisions for the next cycle

$$\mathcal{U}_A = \{\text{Scale Dispensers (A1)},\ \text{Revise Training (A2)},\ \text{Replace Posters (A3)}\}.$$

Decision	Description	T_A	I_A	F_A
A1	Scale P2 hospital-wide (high observed effect, low uncertainty)	0.82	0.12	0.06
A2	Refine P1 with unit-specific scenarios and micro–quizzes	0.65	0.20	0.15
A3	Retire P3; deploy QR-based micro-lessons instead	0.55	0.25	0.20

The *Check* phase shows moderate support for meeting the compliance target (C1), limited evidence on the proxy outcome (C2), and strong audit completeness (C3). Accordingly, *Act* prioritizes scaling dispenser density (A1) because its neutrosophic truth degree is high and indeterminacy is low, schedules a targeted revision of training (A2), and replaces low-yield posters with just-in-time micro-lessons (A3). The next PDCA iteration then re-estimates (T, I, F) from fresh audits, closing the improvement loop.

4.9 DMAIC Cycle with Neutrosophic Sets

DMAIC is a central Six Sigma methodology [173, 174] for systematic process improvement [175]. It proceeds through five phases—Define, Measure, Analyze, Improve, and Control—to optimize performance [175–180]. Neutrosophic and fuzzy adaptations have also been explored [181–183]. We formalize a neutrosophic extension below.

Definition 4.23 (Neutrosophic DMAIC Cycle)**.** [88] With $T, I, F \in [0, 1]$ denoting truth, indeterminacy, and falsity degrees, the neutrosophic DMAIC cycle is specified by:

1. *Define (D).* A neutrosophic set on definition elements,

$$D_f = \left\{ (x, T_{D_f}(x), I_{D_f}(x), F_{D_f}(x)) \;\middle|\; x \in \text{Definition Elements} \right\},$$

   where $T_{D_f}(x)$ measures definition accuracy, $I_{D_f}(x)$ the uncertainty in defining x, and $F_{D_f}(x)$ the degree of inaccuracy.

2. *Measure (M).* A neutrosophic set on measurement elements,

$$M = \left\{ (y, T_M(y), I_M(y), F_M(y)) \;\middle|\; y \in \text{Measurement Elements} \right\},$$

   where $T_M(y)$ expresses measurement reliability, $I_M(y)$ the uncertainty of the measurement process, and $F_M(y)$ the unreliability degree.

3. *Analyze (A).* A neutrosophic set on analysis elements,

$$A_n = \left\{ (z, T_{A_n}(z), I_{A_n}(z), F_{A_n}(z)) \;\middle|\; z \in \text{Analysis Elements} \right\},$$

   where $T_{A_n}(z)$ indicates correctness of analytical results, $I_{A_n}(z)$ the analysis uncertainty, and $F_{A_n}(z)$ the incorrectness degree.

4. *Improve (I).* A neutrosophic set on improvement actions,

$$I_m = \left\{ (w, T_{I_m}(w), I_{I_m}(w), F_{I_m}(w)) \;\middle|\; w \in \text{Improvement Actions} \right\},$$

   where $T_{I_m}(w)$ reflects improvement success, $I_{I_m}(w)$ the uncertainty regarding effectiveness, and $F_{I_m}(w)$ the failure degree.

5. *Control (C).* A neutrosophic set on control elements,

$$C_t = \{ (v, T_{C_t}(v), I_{C_t}(v), F_{C_t}(v)) \mid v \in \text{Control Elements} \},$$

where $T_{C_t}(v)$ quantifies control effectiveness, $I_{C_t}(v)$ the uncertainty of the control process, and $F_{C_t}(v)$ the ineffectiveness degree.

Example 4.24 (A Real-World Neutrosophic DMAIC in Electronics Assembly)**.** A smartphone camera–module assembly line exhibits excessive rework due to solder-bridge defects on fine-pitch pads. The goal is to reduce defects from 3,200 to 800 DPMO. For each DMAIC phase we specify a neutrosophic set over concrete elements. Here $T(\cdot), I(\cdot), F(\cdot) \in [0, 1]$ denote degrees of truth (support), indeterminacy (uncertainty), and falsity (counter-evidence), respectively; in neutrosophic modeling they need not sum to 1.

Define (D_f): problem framing and CTQs

$$\mathcal{U}_{D_f} = \{\text{Problem Statement (Df1)}, \text{CTQ Metric (Df2)}, \text{Customer Impact (Df3)}\}.$$

Element	Description	T_{D_f}	I_{D_f}	F_{D_f}
Df1	Solder bridges on U203 pads cause rework and scrap	0.90	0.07	0.03
Df2	CTQ: DPMO on U203 solder joints, baseline 3,200	0.85	0.10	0.05
Df3	Field-return risk and shipment delays quantified	0.70	0.20	0.10

Measure (M): data integrity and capability

$$\mathcal{U}_M = \{\text{Sampling Plan (M1)}, \text{Gauge R\&R (M2)}, \text{Process Capability (M3)}\}.$$

Element	Description	T_M	I_M	F_M
M1	Shift-balanced sampling across lots and lines	0.78	0.15	0.07
M2	AOI classification gauge R&R < 10% (pilot)	0.65	0.25	0.10
M3	C_{pk} on paste height near spec limit	0.50	0.30	0.20

Analyze (A_n): root-cause hypotheses

$$\mathcal{U}_{A_n} = \{\text{Stencil Wear (An1)}, \text{Squeegee Pressure (An2)}, \text{Placement Offset (An3)}\}.$$

Hypothesis	Rationale / Evidence	T_{A_n}	I_{A_n}	F_{A_n}
An1	Aperture rounding after > 40k prints (microscopy)	0.72	0.18	0.10
An2	Over-pressure $\Rightarrow$ paste smear at pad edges	0.68	0.22	0.10
An3	Pick-and-place y-offset spike on line 3 (logs)	0.55	0.30	0.15

Improve (I_m): countermeasures (pilot)

$$\mathcal{U}_{I_m} = \{\text{Stencil Refresh (Im1)}, \text{Pressure Window (Im2)}, \text{Camera Recal. (Im3)}\}.$$

Action	Description	T_{I_m}	I_{I_m}	F_{I_m}
Im1	Replace stencil at 30k prints; clean every 5 boards	0.80	0.12	0.08
Im2	Set squeegee pressure control limits; alarm at drift	0.75	0.15	0.10
Im3	Weekly vision calibration on line 3	0.58	0.27	0.15

Control (C_t): sustainment plan

$$\mathcal{U}_{C_t} = \{\text{SPC Charts (Ct1)}, \text{Poka-Yoke Sensor (Ct2)}, \text{Layered Audit (Ct3)}\}.$$

Control	Description	T_{C_t}	I_{C_t}	F_{C_t}
Ct1	$X-R$ on paste height and pressure with rules (WECO)	0.82	0.10	0.08
Ct2	Squeegee-load interlock prevents over-pressure start	0.77	0.15	0.08
Ct3	Daily layered process audits with log review	0.70	0.20	0.10

Measurement shows marginal capability and moderate gauge uncertainty. Analysis favors stencil wear and pressure drift as primary drivers. Piloted improvements (Im1–Im2) exhibit higher neutrosophic truth and lower indeterminacy than Im3, so they are prioritized for scale-up, while Control elements Ct1–Ct3 institutionalize the gains. Subsequent monthly reviews update the neutrosophic degrees using fresh SPC and AOI data; if targets remain unmet, the cycle iterates with refined hypotheses and actions.

4.10 SWOT Analysis with Neutrosophic Sets

SWOT is a strategic tool for assessing internal and external factors of a project or organization by cataloging *Strengths*, *Weaknesses*, *Opportunities*, and *Threats* [184–190]. It is used across business, education [185], and healthcare [191], with fuzzy/neutrosophic variants also studied [192–194]. We give a neutrosophic formulation.

Definition 4.25 (Neutrosophic SWOT)**.** [88] Let $T, I, F \in [0, 1]$ represent truth, indeterminacy, and falsity degrees. Define:

1. *Strengths (S).*
$$S = \{ (x, T_S(x), I_S(x), F_S(x)) \mid x \in \text{Strength Elements} \},$$
where $T_S(x)$ is the extent x constitutes a strength, $I_S(x)$ the uncertainty in that assessment, and $F_S(x)$ the extent x is not a strength.

2. *Weaknesses (W).*
$$W = \{ (y, T_W(y), I_W(y), F_W(y)) \mid y \in \text{Weakness Elements} \},$$
where $T_W(y)$ captures how much y is a weakness, $I_W(y)$ the uncertainty of that classification, and $F_W(y)$ the degree to which y is not a weakness.

3. *Opportunities (O).*
$$O = \{ (z, T_O(z), I_O(z), F_O(z)) \mid z \in \text{Opportunity Elements} \},$$
where $T_O(z)$ indicates that z is an opportunity, $I_O(z)$ the uncertainty of this judgment, and $F_O(z)$ the degree to which z is not an opportunity.

4. *Threats (T).*
$$T = \{ (w, T_T(w), I_T(w), F_T(w)) \mid w \in \text{Threat Elements} \},$$
where $T_T(w)$ measures the extent w is a threat, $I_T(w)$ the uncertainty in that determination, and $F_T(w)$ the extent w is not a threat.

Example 4.26 (A Real-World Neutrosophic SWOT: Community Hospital Telemedicine)**.** A mid-size community hospital plans to expand a *telemedicine* service line. We model each SWOT quadrant as a neutrosophic set over concrete elements. Here $T(\cdot), I(\cdot), F(\cdot) \in [0, 1]$ denote degrees of truth (support), indeterminacy (uncertainty), and falsity (counter-evidence), respectively; in neutrosophic modeling they need not sum to 1.

Strengths (S):
$$\mathcal{U}_S = \{\text{EHR Integration (S1)}, \text{IT Expertise (S2)}, \text{Brand Trust (S3)}\}.$$

Element	Description	T_S	I_S	F_S
S1	Existing EHR interfaces support secure video visits and e-prescribing	0.82	0.10	0.08
S2	In-house IT team experienced with HIPAA-compliant cloud deployments	0.75	0.15	0.10
S3	High local reputation and strong PCP referral network	0.68	0.22	0.10

Weaknesses (W):
$$\mathcal{U}_W = \{\text{Patient Connectivity (W1)}, \text{Clinician Adoption (W2)}, \text{Scheduling Flow (W3)}\}.$$

Element	Description	T_W	I_W	F_W
W1	Rural patients have limited broadband or device access	0.80	0.12	0.08
W2	A subset of clinicians prefer in-person visits; training needed	0.62	0.25	0.13
W3	Manual scheduling causes bottlenecks at peak hours	0.55	0.30	0.15

Opportunities (O):

$$\mathcal{U}_O = \{\text{Payer Policies (O1)}, \text{Employer Partnerships (O2)}, \text{Remote Monitoring (O3)}\}.$$

Element	Description	T_O	I_O	F_O
O1	Expanded telehealth reimbursement for primary and behavioral care	0.78	0.15	0.07
O2	Onsite/virtual care bundles for local employers	0.66	0.24	0.10
O3	Chronic disease programs (BP, glucose, COPD) via home devices	0.72	0.20	0.08

Threats (T):

$$\mathcal{U}_T = \{\text{Platform Competition (T1)}, \text{Regulatory Shifts (T2)}, \text{Cybersecurity (T3)}\}.$$

Element	Description	T_T	I_T	F_T
T1	National direct-to-consumer telehealth brands entering the region	0.70	0.20	0.10
T2	Potential changes to privacy and cross-state licensure rules	0.65	0.28	0.07
T3	Rising incidence of healthcare ransomware attacks	0.74	0.18	0.08

High T_S for EHR integration (S1) and IT expertise (S2) indicate strong technical readiness, while W1 and W2 reveal adoption risks that carry moderate $I(\cdot)$. Opportunities O1 and O3 score high on T_O, supporting early service lines (e.g., primary care and chronic management). Threats T1–T3 suggest prioritizing differentiation (care continuity), compliance monitoring, and layered cybersecurity controls. Periodic updates of (T, I, F) using utilization, patient-experience, and incident data keep the analysis current and actionable.

4.11 OODA Cycle with Neutrosophic Sets

The OODA (Observe–Orient–Decide–Act) cycle is a decision framework for responding effectively in fast-changing, competitive settings [195–200]. It proceeds by (i) observing the environment, (ii) orienting using context, (iii) deciding, and (iv) acting. Below we present an extension in which each stage is modeled by a Neutrosophic Set, capturing truth, indeterminacy, and falsity.

Definition 4.27 (Neutrosophic OODA Loop)**.** [88] Let $T(\cdot), I(\cdot), F(\cdot) \in [0, 1]$ denote, respectively, degrees of truth, indeterminacy, and falsity. The neutrosophic version of OODA comprises four stages:

1. *Observe (O).* A neutrosophic set O_b over the universe of observation elements:

$$O_b = \left\{ (x, T_{O_b}(x), I_{O_b}(x), F_{O_b}(x)) \;\middle|\; x \in \text{Observation Elements} \right\},$$

   where $T_{O_b}(x)$ is the degree to which x is correctly observed, $I_{O_b}(x)$ quantifies uncertainty in observing x, and $F_{O_b}(x)$ measures the degree of misobservation of x.

2. *Orient (O).* A neutrosophic set O_r over orientation elements:

$$O_r = \left\{ (y, T_{O_r}(y), I_{O_r}(y), F_{O_r}(y)) \;\middle|\; y \in \text{Orientation Elements} \right\},$$

   where $T_{O_r}(y)$ captures how correct the orientation is, $I_{O_r}(y)$ the uncertainty of that orientation, and $F_{O_r}(y)$ the degree to which the orientation is incorrect.

3. *Decide (D).* A neutrosophic set D_c over decision elements:

$$D_c = \left\{ (z, T_{D_c}(z), I_{D_c}(z), F_{D_c}(z)) \;\middle|\; z \in \text{Decision Elements} \right\},$$

   where $T_{D_c}(z)$ measures decision correctness, $I_{D_c}(z)$ the uncertainty of the decision, and $F_{D_c}(z)$ the degree of decision incorrectness.

4. *Act (A).* A neutrosophic set A_c over action elements:

$$A_c = \{ (w, T_{A_c}(w), I_{A_c}(w), F_{A_c}(w)) \mid w \in \text{Action Elements} \},$$

where $T_{A_c}(w)$ reflects action effectiveness, $I_{A_c}(w)$ the uncertainty of that action, and $F_{A_c}(w)$ the degree to which the action is ineffective.

Example 4.28 (A Real-World Neutrosophic OODA: Autonomous Driving in Heavy Rain)**.** An autonomous vehicle (AV) approaches a congested urban intersection during heavy rain. We model one OODA iteration using neutrosophic sets. For each element, the triple $(T, I, F) \in [0, 1]^3$ denotes degrees of truth (support), indeterminacy (uncertainty), and falsity (counter–evidence); in neutrosophic modeling these need not sum to 1.

Observe (O_b). Sensor-level observations:

$$O_b = \{(x, T_{O_b}(x), I_{O_b}(x), F_{O_b}(x))\}.$$

Observation x	**Description**	T_{O_b}	I_{O_b}	F_{O_b}
x_1	Radar: strong return consistent with *stopped vehicle* at ≈35 m	0.82	0.12	0.06
x_2	Camera: lane markings *degraded* by rain and glare	0.76	0.18	0.06
x_3	LiDAR: reflective puddles causing partial ghosting	0.55	0.30	0.15
x_4	V2X: broadcast "hard braking ahead"	0.88	0.08	0.04

Orient (O_r). Situation hypotheses informed by O_b:

$$O_r = \{(y, T_{O_r}(y), I_{O_r}(y), F_{O_r}(y))\}.$$

Hypothesis y	**Description**	T_{O_r}	I_{O_r}	F_{O_r}
y_1	Object ahead is a *stopped car*, not a puddle artifact	0.80	0.15	0.05
y_2	*Right lane occupancy is low* (overtake may be feasible)	0.60	0.25	0.15
y_3	*Lane-keeping risk is medium* due to weak markings	0.72	0.20	0.08

Decide (D_c). Candidate decisions:

$$D_c = \{(z, T_{D_c}(z), I_{D_c}(z), F_{D_c}(z))\}.$$

Decision z	**Description**	T_{D_c}	I_{D_c}	F_{D_c}
z_1	*Brake in lane* to safe headway (decel ≈0.3g)	0.78	0.15	0.07
z_2	*Change to right lane* and pass the obstacle	0.54	0.28	0.18
z_3	*Pull over to shoulder* and stop	0.46	0.26	0.28

One possible selector. For illustration, define a simple neutrosophic score

$$\text{Score}(z) := T_{D_c}(z) - \big(F_{D_c}(z) + \alpha\, I_{D_c}(z)\big), \qquad \alpha = 0.5.$$

Then

$$\text{Score}(z_1) = 0.78 - (0.07 + 0.5 \cdot 0.15) = 0.635,$$
$$\text{Score}(z_2) = 0.54 - (0.18 + 0.5 \cdot 0.28) = 0.220,$$
$$\text{Score}(z_3) = 0.46 - (0.28 + 0.5 \cdot 0.26) = 0.050,$$

so z_1 is selected.

Act (A_c). Concrete control actions corresponding to the top decision:

$$A_c = \{(w, T_{A_c}(w), I_{A_c}(w), F_{A_c}(w))\}.$$

Action w	**Description**	T_{A_c}	I_{A_c}	F_{A_c}
w_1	Apply 3.0 m/s^2 braking; hold lane center with damped steering	0.80	0.14	0.06
w_2	Signal and shift to right lane with mild acceleration	0.50	0.30	0.20
w_3	Hazard lights; decelerate and move to shoulder	0.45	0.25	0.30

Using the same selector,

$$\text{Score}(w_1) = 0.80 - (0.06 + 0.5 \cdot 0.14) = 0.67, \quad \text{Score}(w_2) = 0.15, \quad \text{Score}(w_3) = 0.025,$$

so the controller executes w_1.

Looping. After executing w_1, new observations (e.g., increased headway, reduced V2X alerts) update (T, I, F) in O_b, which propagates to O_r, D_c, and A_c for the next, faster OODA iteration. This illustrates how neutrosophic degrees capture sensor noise (rain glare), model uncertainty (lane occupancy), and safety trade-offs in real-time autonomous driving.

4.12 Neutrosophic Porter's Five Forces Analysis

Porter's Five Forces is a classic tool for evaluating industry competition through five factors: rivalry among incumbents, buyer power, supplier power, threat of substitutes, and threat of new entrants [201–203]. Related formulations exist for Fuzzy and Neutrosophic settings [204]. We formalize a neutrosophic extension below.

Definition 4.29 (Neutrosophic Five Forces)**.** [88] Let each force be represented as a Neutrosophic Set to encode truth, indeterminacy, and falsity about its contributing factors:

1. *Threat of New Entrants (N).*
$$N = \{ (x, T_N(x), I_N(x), F_N(x)) \mid x \in \text{New Entrant Factors} \},$$
where $T_N(x)$ is the degree to which x raises entry threat, $I_N(x)$ the uncertainty about x's impact, and $F_N(x)$ the degree to which x does not raise entry threat.

2. *Bargaining Power of Suppliers (S).*
$$S = \{ (y, T_S(y), I_S(y), F_S(y)) \mid y \in \text{Supplier Factors} \},$$
where $T_S(y)$ measures the extent y strengthens supplier power, $I_S(y)$ the uncertainty of its effect, and $F_S(y)$ the degree to which y does not strengthen supplier power.

3. *Bargaining Power of Buyers (B).*
$$B = \{ (z, T_B(z), I_B(z), F_B(z)) \mid z \in \text{Buyer Factors} \},$$
where $T_B(z)$ indicates how much z increases buyer power, $I_B(z)$ its associated uncertainty, and $F_B(z)$ the degree to which z does not increase buyer power.

4. *Threat of Substitutes (U).*
$$U = \{ (w, T_U(w), I_U(w), F_U(w)) \mid w \in \text{Substitute Factors} \},$$
where $T_U(w)$ is the degree to which w heightens substitute threat, $I_U(w)$ the uncertainty of that influence, and $F_U(w)$ the degree to which w does not heighten substitute threat.

5. *Industry Rivalry (R).*
$$R = \{ (v, T_R(v), I_R(v), F_R(v)) \mid v \in \text{Rivalry Factors} \},$$
where $T_R(v)$ measures how strongly v intensifies rivalry, $I_R(v)$ the uncertainty about v's effect, and $F_R(v)$ the degree to which v does not intensify rivalry.

Example 4.30 (A Neutrosophic Five Forces Analysis: Telemedicine Startup in a Metropolitan Region)**.** Consider a seed-stage telemedicine startup offering same-day video visits in a large city. Each force is modeled by a Neutrosophic Set over its factors, with triples $(T, I, F) \in [0, 1]^3$ denoting, respectively, the degree that the factor *supports* the force, its *indeterminacy*, and the degree it *does not support* the force. For quick prioritization we use a simple neutrosophic score
$$\text{Score}(x) = T(x) - \left(F(x) + \tfrac{1}{2} I(x)\right), \qquad \text{ForceScore} = \sum_{x \in \text{factors}} \text{Score}(x).$$

(1) Threat of New Entrants $N = \{(x, T_N, I_N, F_N)\}$

Factor	T	I	F	Score
Cloud infrastructure lowers barriers	0.78	0.12	0.10	$0.78 - (0.10 + 0.06) = \mathbf{0.62}$
Licensing/credentialing requirements	0.20	0.20	0.60	$0.20 - (0.60 + 0.10) = \mathbf{-0.50}$
App-store/publishing ease	0.60	0.20	0.20	$0.60 - (0.20 + 0.10) = \mathbf{0.30}$
		ForceScore		$\mathbf{0.62 - 0.50 + 0.30 = 0.42}$

(2) Supplier Power $S = \{(y, T_S, I_S, F_S)\}$ (physicians, platforms)

Factor	T	I	F	Score
Scarcity of board-certified physicians	0.75	0.15	0.10	$0.75 - (0.10 + 0.08) = \mathbf{0.575}$
EHR API/vendor lock-in	0.70	0.20	0.10	$0.70 - (0.10 + 0.10) = \mathbf{0.50}$
Commodity video infrastructure	0.25	0.25	0.50	$0.25 - (0.50 + 0.13) = \mathbf{-0.375}$
		ForceScore		$\mathbf{0.575 + 0.50 - 0.375 = 0.70}$

(3) Buyer Power $B = \{(z, T_B, I_B, F_B)\}$ (patients, employers/insurers)

Factor	T	I	F	Score
Employer group contracts concentrate demand	0.72	0.18	0.10	$0.72 - (0.10 + 0.09) = \mathbf{0.53}$
Low switching costs between apps	0.80	0.10	0.10	$0.80 - (0.10 + 0.05) = \mathbf{0.65}$
Insurance reimbursement leverage	0.65	0.25	0.10	$0.65 - (0.10 + 0.13) = \mathbf{0.425}$
		ForceScore		$\mathbf{0.53 + 0.65 + 0.425 = 1.605}$

(4) Threat of Substitutes $U = \{(w, T_U, I_U, F_U)\}$

Factor	T	I	F	Score
In-person clinics as alternative	0.70	0.15	0.15	$0.70 - (0.15 + 0.08) = \mathbf{0.475}$
Retail clinics offering telehealth	0.68	0.20	0.12	$0.68 - (0.12 + 0.10) = \mathbf{0.46}$
Asynchronous chatbots/Q&A	0.55	0.25	0.20	$0.55 - (0.20 + 0.13) = \mathbf{0.225}$
		ForceScore		$\mathbf{0.475 + 0.46 + 0.225 = 1.16}$

(5) Industry Rivalry $R = \{(v, T_R, I_R, F_R)\}$

Factor	T	I	F	Score
Many VC-backed entrants	0.82	0.10	0.08	$0.82 - (0.08 + 0.05) = \mathbf{0.69}$
Escalating marketing spend	0.78	0.12	0.10	$0.78 - (0.10 + 0.06) = \mathbf{0.62}$
Niche specialization (differentiation)	0.30	0.20	0.50	$0.30 - (0.50 + 0.10) = \mathbf{-0.30}$
		ForceScore		$\mathbf{0.69 + 0.62 - 0.30 = 1.01}$

With the above (illustrative) assessments, the most constraining forces are *Buyer Power* (1.605) and *Threat of Substitutes* (1.16), followed by *Industry Rivalry* (1.01). *Supplier Power* is moderate (0.70), and *Threat of New Entrants* is comparatively lower (0.42) due to licensing frictions, despite low technical barriers. The neutrosophic treatment makes the role of indeterminacy explicit in each factor's contribution.

4.13 Neutrosophic Storytelling

Storytelling is the process of conveying information, values, or experiences through structured narratives, fostering emotional engagement and facilitating knowledge transfer [205–209]. This concept is extended using Neutrosophic Logic, leading to the development of Neutrosophic Storytelling. The definitions and associated concepts are provided below.

Definition 4.31 (Neutrosophic Storytelling)**.** [88] *Neutrosophic Storytelling* extends traditional storytelling by integrating neutrosophic logic into the narrative process, capturing uncertainty, indeterminacy, and falsity in audience comprehension and value transmission. It is defined as a tuple:

$$S_N = (N, R, V^N, A^N, T, C^N),$$

where:

- $N = \{n_1, n_2, \ldots, n_m\}$: A sequence of narrative events n_i, where each n_i represents a discrete element of the story.
- $R : N \times N \to \mathcal{R}$: The *relation function*, mapping pairs of events (n_i, n_j) to a set of relationships $\mathcal{R}$, such as causality, sequence, or thematic links.
- $V^N : N \to [0, 1]^3$: The *neutrosophic value function*, assigning a triplet $V^N(n_i) = (T_{n_i}, I_{n_i}, F_{n_i})$ to each event n_i, representing its truth (T), indeterminacy (I), and falsity (F).
- $A^N : E \times N \to [0, 1]^3$: The *neutrosophic audience comprehension function*, where $A^N(e, n_i) = (T_{e,n_i}, I_{e,n_i}, F_{e,n_i})$ measures the audience member e's degree of understanding, uncertainty, and misunderstanding for event n_i.
- $T = \{t_1, t_2, \ldots, t_p\}$: A time sequence over which the narrative is delivered.
- $C^N : N \to K$: The *neutrosophic knowledge content function*, mapping each event n_i to a knowledge element $k \in K$, with truth, indeterminacy, and falsity components.

The total neutrosophic impact I^N of storytelling for an audience E is defined as:

$$I^N = \sum_{n_i \in N} \sum_{e \in E} \left(T_{e,n_i} - F_{e,n_i}\right) \cdot V^N(n_i) \cdot C^N(n_i).$$

The storytelling process is deemed effective if:

$$I^N \geq I^N_{\text{target}},$$

where I^N_{target} is the minimum desired neutrosophic impact threshold.

Example 4.32 (Neutrosophic Storytelling for corporate safety training)**.** A manufacturing company wants to reduce near–miss incidents on the shop floor. Instead of sending only a PDF manual, the trainer tells a short story about "Ken," a senior operator, who avoided a serious accident because he followed the lockout–tagout (LOTO) rule even under time pressure.

Let the narrative events be

$$N = \{n_1, n_2, n_3, n_4\},$$

where

- n_1: "Morning briefing about a rush order."
- n_2: "Ken notices a jam in the conveyor."
- n_3: "Ken decides to apply LOTO before touching the machine."
- n_4: "Supervisor praises Ken and explains why LOTO saved time overall."

The relations are

$$R(n_1, n_2) = \text{causes (rush leads to risky situation)}, \quad R(n_2, n_3) = \text{decision}, \quad R(n_3, n_4) = \text{consequence/explanation}.$$

Suppose the trainer assigns the neutrosophic value of each event (story–wise importance and factuality) as

$$V^N(n_1) = (0.70, 0.20, 0.10), \quad V^N(n_2) = (0.80, 0.10, 0.10), \quad V^N(n_3) = (0.95, 0.03, 0.02), \quad V^N(n_4) = (0.85, 0.10, 0.05).$$

Here $V^N(n_3)$ is highest because the core message is "apply LOTO even under pressure."

There are three audience members $E = \{e_1, e_2, e_3\}$: a new hire (e_1), a contractor (e_2), and a senior but distracted worker (e_3). Their neutrosophic comprehension for n_3 is observed right after the session:

$$A^N(e_1, n_3) = (0.90, 0.05, 0.05), \quad A^N(e_2, n_3) = (0.65, 0.25, 0.10), \quad A^N(e_3, n_3) = (0.50, 0.30, 0.20).$$

This means: the new hire almost fully got the point; the contractor is unsure how LOTO applies to his subcontract scope (high indeterminacy); the senior worker partially misunderstood, thinking LOTO is "only for maintenance".

Let the neutrosophic knowledge content be

$$C^N(n_1) = k_{\text{urgency}}, \quad C^N(n_2) = k_{\text{hazard_detection}}, \quad C^N(n_3) = k_{\text{LOTO_rule}}, \quad C^N(n_4) = k_{\text{benefit}},$$

and suppose each k is normalized to 1 for simplicity.

Then the (simplified) neutrosophic impact of the story is

$$\begin{aligned} I^N &= \sum_{n_i \in N} \sum_{e \in E} (T_{e,n_i} - F_{e,n_i}) \cdot T_{n_i} \\ &\approx \underbrace{(0.90 - 0.05) \cdot 0.95}_{e_1, n_3} + \underbrace{(0.65 - 0.10) \cdot 0.95}_{e_2, n_3} + \underbrace{(0.50 - 0.20) \cdot 0.95}_{e_3, n_3} + (\text{smaller contributions from } n_1, n_2, n_4). \end{aligned}$$

Numerically,

$$(0.85) \cdot 0.95 \approx 0.8075, \quad (0.55) \cdot 0.95 \approx 0.5225, \quad (0.30) \cdot 0.95 \approx 0.285,$$

so, ignoring the smaller terms,

$$I^N \gtrsim 0.8075 + 0.5225 + 0.285 = 1.615.$$

If the safety officer set

$$I^N_{\text{target}} = 1.50,$$

then

$$I^N \geq I^N_{\text{target}},$$

and the session is declared *neutrosophically effective*: the true absorption of the central rule (LOTO) by the group, *despite* pockets of indeterminacy and some misunderstanding, has reached the required threshold.

Operationally, the officer can now focus on the subgroup with high indeterminacy or falsity (here e_2 and e_3) and provide a short follow–up micro–story, e.g. "LOTO for contractors" or "LOTO for quick fixes," thus lowering I_{e_2,n_3} and F_{e_3,n_3} in a second round.

4.14 Neutrosophic Work-Life Balance

Work–life balance denotes the deliberate allocation of limited human resources (time, energy, attention) between occupational duties and non–work domains so that health, performance, and continuity are maintained [210–216]. When this idea is lifted to a neutrosophic setting, every evaluation is decomposed into a degree that is confirmed, a degree that is uncertain, and a degree that is refuted. What follows is a prototype model; further calibration, domain constraints, and empirical validation can be added in subsequent studies.

Definition 4.33 (Neutrosophic Work-Life Balance)**.** [88] A *Neutrosophic Work–Life Balance (NWLB) system* is a septuple

$$\mathcal{NWLB} = (W, L, T^N, U^N, C^N, R^N, \mathcal{S}^N),$$

with the following components.

- $W = \{w_1, \ldots, w_n\}$: finite family of work–related activities (reports, supervision, meetings, teaching, on–call, projects).
- $L = \{l_1, \ldots, l_m\}$: finite family of non–work activities (family time, sleep, exercise, hobbies, caregiving).
- $T^N : (W \cup L) \to [0,1]^3$ is the *neutrosophic time–allocation map*,

  $$T^N(x) = \big(T_T(x),\, T_I(x),\, T_F(x)\big),$$

  where $T_T(x)$ is the confirmed (actually scheduled/used) proportion of time for x, $T_I(x)$ captures ambiguity (interruption, overtime, unscheduled care work), and $T_F(x)$ records the rejected/illusory part of the supposed time for x.
- $U^N : (W \cup L) \to \mathbb{R}^3$ is the *neutrosophic utility map*,

  $$U^N(x) = \big(U_T(x),\, U_I(x),\, U_F(x)\big),$$

  where U_T measures realized benefit (income, career progress, wellbeing), U_I covers unclear or delayed benefits, and U_F refers to benefit that was expected but did not materialize.
- $C^N : (W \cup L) \to \mathbb{R}^3$ is the *neutrosophic cost map*,

  $$C^N(x) = \big(C_T(x),\, C_I(x),\, C_F(x)\big),$$

  where C_T is the clearly perceived burden (fatigue, stress, commute), C_I is uncertain or context–dependent burden (unexpected calls, emotional labor), and C_F is overestimated or cancelled burden.
- $R^N : (W \cup L) \to \mathbb{R}^3$ is the *neutrosophic recovery map*,

  $$R^N(x) = \big(R_T(x),\, R_I(x),\, R_F(x)\big),$$

  representing, respectively, actual restoration, partly unknown restoration (e.g. micro–breaks), and restoration that was planned but did not occur.
- $\mathcal{S}^N = (S_W^N, S_L^N, \Omega^N)$ is the *neutrosophic sustainability profile*, where S_W^N aggregates the work side, S_L^N aggregates the life side, and Ω^N is a global feasibility band fixed by the person or the organization.

Notation 4.34 (Aggregated neutrosophic scores)**.** *Assume that the decision maker primarily evaluates the* truth *parts of the utilities and costs. Then the work–side and life–side balances are*

$$S_W^N \;=\; \sum_{w_i \in W} \big(U_T(w_i) - C_T(w_i)\big), \qquad S_L^N \;=\; \sum_{l_j \in L} \big(U_T(l_j) + R_T(l_j) - C_T(l_j)\big).$$

The overall neutrosophic balance is

$$\mathcal{S}^N \;=\; S_W^N + S_L^N.$$

A personal (or institutional) admissible interval is fixed as

$$\Omega^N \in \big[\Omega^N_{\min},\, \Omega^N_{\max}\big],$$

which represents the zone in which the current pattern of work and life is regarded as sustainable.

Definition 4.35 (Neutrosophic balance condition)**.** A schedule (T^N, U^N, C^N, R^N) is said to *achieve neutrosophic work–life balance* if

$$\mathcal{S}^N \;\in\; \big[\Omega^N_{\min},\, \Omega^N_{\max}\big].$$

Equivalently, the confirmed benefits of work and life, together with confirmed recovery, are high enough to compensate confirmed burdens.

Example 4.36 (Neutrosophic balance condition: mid–career researcher with caregiving duties)**.** Consider a researcher, Alice, who works at a university three days on campus and two days remotely. Let

$$W = \{\text{paper}, \text{teaching}, \text{admin}\}, \qquad L = \{\text{family}, \text{sleep}, \text{exercise}\}.$$

Suppose the confirmed (truth) utilities and costs are as follows:

x	$U_T(x)$	$C_T(x)$	$R_T(x)$
paper	0.80	0.30	0
teaching	0.60	0.25	0
admin	0.30	0.20	0
family	0.70	0.05	0.25
sleep	0.50	0	0.40
exercise	0.40	0.05	0.30

Then the work–side balance is

$$S_W^N = (0.80 - 0.30) + (0.60 - 0.25) + (0.30 - 0.20) = 0.50 + 0.35 + 0.10 = 0.95.$$

The life–side balance is

$$\begin{aligned} S_L^N &= (0.70 + 0.25 - 0.05) + (0.50 + 0.40 - 0) + (0.40 + 0.30 - 0.05) \\ &= (0.90) + (0.90) + (0.65) = 2.45. \end{aligned}$$

Hence the overall neutrosophic balance is

$$\mathcal{S}^N = S_W^N + S_L^N = 0.95 + 2.45 = 3.40.$$

Assume that Alice (or the HR policy) sets the admissible sustainability band as

$$\Omega_{\min}^N = 3.0, \qquad \Omega_{\max}^N = 3.6.$$

Since

$$\mathcal{S}^N = 3.40 \in [3.0, 3.6],$$

the *neutrosophic balance condition* is satisfied: confirmed gains from work and life, plus confirmed recovery, offset confirmed burdens. The indeterminacy parts T_I, U_I, C_I, R_I may exist (e.g. unexpected child care, sudden faculty meeting), but they do not push the system out of the sustainability band.

Definition 4.37 (Neutrosophic imbalance)**.** If

$$\mathcal{S}^N \notin \left[\Omega_{\min}^N, \Omega_{\max}^N\right],$$

then the configuration is called a *neutrosophic work–life imbalance*. In this situation, at least one of the following holds: (i) confirmed work utility is too low, (ii) confirmed life/recovery is too low, (iii) confirmed costs are too high, (iv) indeterminacy components are so large that the agent cannot guarantee sustainability.

Example 4.38 (Neutrosophic imbalance: IT project lead during product launch)**.** Let Bob be an IT project lead in the launch week of a cloud product. His activities are

$$W = \{\text{oncall}, \text{deploy}, \text{client_calls}\}, \qquad L = \{\text{family_dinner}, \text{sleep}\}.$$

Because of launch pressure, the confirmed costs rise and recovery drops. Assume

x	$U_T(x)$	$C_T(x)$	$R_T(x)$
oncall	0.70	0.55	0
deploy	0.80	0.50	0
client_calls	0.60	0.40	0
family_dinner	0.50	0.10	0.10
sleep	0.30	0	0.20

Then

$$S_W^N = (0.70 - 0.55) + (0.80 - 0.50) + (0.60 - 0.40) = 0.15 + 0.30 + 0.20 = 0.65.$$

For life:

$$\begin{aligned} S_L^N &= (0.50 + 0.10 - 0.10) + (0.30 + 0.20 - 0) \\ &= (0.50) + (0.50) = 1.00. \end{aligned}$$

Thus

$$\mathcal{S}^N = S_W^N + S_L^N = 0.65 + 1.00 = 1.65.$$

Suppose Bob's usual sustainability band is

$$\Omega^N_{\min} = 2.5, \qquad \Omega^N_{\max} = 3.4.$$

Since

$$\mathcal{S}^N = 1.65 \notin [2.5, 3.4],$$

this week is a *neutrosophic work–life imbalance*. In neutrosophic terms, there is also high indeterminacy:

$$T_I(\text{oncall}) = 0.30, \quad C_I(\text{deploy}) = 0.25,$$

coming from unplanned midnight incidents and unclear client escalation. These uncertain burdens make it impossible to guarantee that future days will enter the admissible band, so the configuration remains imbalanced until either costs are reduced or recovery is increased.

Definition 4.39 (Optimal neutrosophic work-life plan)**.** Among all neutrosophic time allocations $\{T^N(x)\}_{x\in W\cup L}$ satisfying the total–time constraint

$$\sum_{x\in W\cup L} T_T(x) \;=\; T_{\text{total}},$$

an *optimal* plan is any solution of

$$\mathcal{NWLB}^* \;\in\; \underset{\{T^N(x)\}}{\arg\max}\big(S^N_W + S^N_L\big) \quad \text{subject to} \quad \mathcal{S}^N \in \big[\Omega^N_{\min}, \Omega^N_{\max}\big].$$

This selects allocations that maximize confirmed net benefit while remaining inside the sustainability band.

Example 4.40 (Optimal neutrosophic work–life plan: reallocating hours for a physician)**.** Consider Dr. Carol, a hospital physician who currently allocates in a typical week

$$\text{ward} = 20\text{h}, \quad \text{outpatient} = 10\text{h}, \quad \text{research} = 5\text{h}, \quad \text{family} = 15\text{h}, \quad \text{sleep/exercise} = 18\text{h},$$

with a hard time budget of $T_{\text{total}} = 68$h. From the current pattern, the confirmed scores are

$$S^N_W = 1.10, \qquad S^N_L = 1.60, \qquad \mathcal{S}^N = 2.70.$$

Her target band, negotiated with the department, is

$$[\,\Omega^N_{\min}, \Omega^N_{\max}\,] = [2.8, 3.2].$$

Thus the current plan is slightly below the desired band. She considers the following neutrosophic modification:

- reduce outpatient by 2h (cutting low–utility, high–fatigue appointments),
- increase research by 1h (high long–term U_T),
- increase sleep/exercise by 1h (high R_T, almost zero C_T),
- keep family at 15h but mark $T_I(\text{family}) = 0.10$ to reflect possible school events.

After this reallocation, the confirmed utilities and recoveries change to

$$S^N_W = 1.20, \qquad S^N_L = 1.75, \qquad \mathcal{S}^N = 2.95.$$

Because

$$\mathcal{S}^N = 2.95 \in [2.8, 3.2] \quad \text{and} \quad \sum_x T_T(x) = 68\text{h},$$

this new allocation is an *optimal neutrosophic work–life plan* for Carol in the sense of the model: it maximizes (for this small adjustment set) the confirmed net benefit

$$S^N_W + S^N_L$$

while staying inside the prescribed sustainability band. If in addition the hospital lowers $C_T(\text{ward})$ slightly (e.g. by adding a resident), then a second optimal plan with even higher $\mathcal{S}^N$ becomes feasible.

Remark 4.41 (Specializations)**.** (i) If for every x we impose $T_I(x) = T_F(x) = U_I(x) = U_F(x) = C_I(x) = C_F(x) = R_I(x) = R_F(x) = 0$, then the above model reduces to a classical or fuzzy work–life balance formula in which only confirmed quantities are evaluated.
(ii) If, instead, we allow attribute–driven contradiction degrees between activities (for example, high–stress workdays vs. family obligations), the model can be embedded into a plithogenic work–life balance framework, thereby generalizing both neutrosophic and fuzzy variants.

Chapter 5: Neutrosophic Concepts for Informatics

In this section, we present and discuss neutrosophic concepts relevant to informatics, outlining their definitions, properties, and potential applications.

5.1 Neutrosophic RSA Public-Key Cryptosystem

RSA is a public-key cryptosystem using modular exponentiation with large composite moduli; security relies on practical hardness of integer factorization [217–220]. Neutrosophic RSA extends RSA over idempotent neutrosophic integers, encrypting componentwise under dual moduli, modeling truth, indeterminacy, and falsity simultaneously values [221–224].

Definition 5.1 (Neutrosophic Integers and Modulus)**.** (cf. [224–226]) Let

$$\mathbb{Z}(I) := \{\, a + bI \mid a, b \in \mathbb{Z},\ I^2 = I \,\}$$

with componentwise addition and multiplication

$$(a + bI)(c + dI) = ac + (ad + bc + bd)\, I \qquad (\text{since } I^2 = I).$$

For $X = a + bI$ define the *real* and *total* projections

$$\mathrm{Re}(X) := a, \qquad \mathrm{Tot}(X) := a + b.$$

Fix a *modulus* $M = m + nI$ with $m > 0$ and $m + n > 0$. We declare the congruence

$$X \equiv Y \pmod{M} \quad :\Longleftrightarrow \quad \mathrm{Re}(X) \equiv \mathrm{Re}(Y) \pmod{m},\ \ \mathrm{Tot}(X) \equiv \mathrm{Tot}(Y) \pmod{m+n}.$$

Equivalently, via the ring homomorphism

$$\pi_M : \mathbb{Z}(I) \longrightarrow \mathbb{Z}_m \times \mathbb{Z}_{m+n}, \qquad \pi_M(a + bI) = \big(a \bmod m,\ a + b \bmod (m + n)\big),$$

the class of X modulo M is the pair of classes $\big(\mathrm{Re}(X) \bmod m,\ \mathrm{Tot}(X) \bmod (m + n)\big)$. We write $\gcd(X, M) = 1$ iff $\gcd(\mathrm{Re}(X), m) = \gcd(\mathrm{Tot}(X), m + n) = 1$.

Example 5.2 (Neutrosophic Integers and Modulus: a concrete congruence)**.** Let $M = 6 + 3I$; then $m = 6 > 0$ and $m + n = 9 > 0$. Take $X = 23 + 5I$ and $Y = 35 + 11I$. We have

$$\mathrm{Re}(X) = 23, \quad \mathrm{Tot}(X) = 28, \qquad \mathrm{Re}(Y) = 35, \quad \mathrm{Tot}(Y) = 46.$$

Since $35 \equiv 23 \pmod 6$ and $46 \equiv 28 \pmod 9$, it follows by the definition that

$$X \equiv Y \pmod{M}.$$

(Equivalently, $\pi_M(X) = (23 \bmod 6,\ 28 \bmod 9) = (5, 1) = \pi_M(Y)$.) As a check of the ring law in $\mathbb{Z}(I)$,

$$(23 + 5I)(2 + I) = 46 + (23 + 10 + 5)I = 46 + 38I,$$

using $(a + bI)(c + dI) = ac + (ad + bc + bd)I$ with $I^2 = I$.

Definition 5.3 (Special Neutrosophic Euler Function)**.** For $M = m + nI$ with $m > 0,\ m + n > 0$, define

$$\varphi_s(M) \ := \ \varphi(m) \ + \big(\varphi(m + n) - \varphi(m)\big)\, I.$$

Hence $\mathrm{Re}\big(\varphi_s(M)\big) = \varphi(m)$ and $\mathrm{Tot}\big(\varphi_s(M)\big) = \varphi(m + n)$. If $\gcd(A, M) = 1$, then

$$A^{\,\varphi_s(M)} \ \equiv \ 1 \pmod{M},$$

because this holds componentwise after applying π_M. Moreover, if $\gcd(P, Q) = 1$ (componentwise), then $\varphi_s(PQ) = \varphi_s(P)\,\varphi_s(Q)$.

Example 5.4 (Special Neutrosophic Euler Function: verifying $A^{\varphi_s(M)} \equiv 1$)**.** Let $M = 6+4I$ so that $m = 6$ and $m+n = 10$. Then

$$\varphi_s(M) = \varphi(6) + \big(\varphi(10) - \varphi(6)\big)I = 2 + (4 - 2)I = 2 + 2I.$$

Choose $A = 5 + 2I$. We have $\gcd(\mathrm{Re}(A), m) = \gcd(5, 6) = 1$ and $\gcd(\mathrm{Tot}(A), m + n) = \gcd(7, 10) = 1$, so the hypothesis holds. Under the canonical map $\pi_M : \mathbb{Z}(I) \to \mathbb{Z}_6 \times \mathbb{Z}_{10}$,

$$\mathrm{Re}\big(A^{\,\varphi_s(M)}\big) \equiv 5^{\,2} \equiv 25 \equiv 1 \pmod 6, \qquad \mathrm{Tot}\big(A^{\,\varphi_s(M)}\big) \equiv 7^{\,2+2} = 7^4 \equiv 2401 \equiv 1 \pmod{10}.$$

Hence $A^{\,\varphi_s(M)} \equiv 1 \pmod M$, as claimed by the definition.

Definition 5.5 (Neutrosophic RSA Public-Key Cryptosystem)**.** [221–224] **Key generation.** Choose $P = a + bI,\ Q = c + dI \in \mathbb{Z}(I)$ with $a > 0,\ a + b > 0,\ c > 0,\ c + d > 0$ and

$$\gcd(a, c) = 1, \qquad \gcd(a + b, c + d) = 1 \quad (\text{i.e. } \gcd(P, Q) = 1).$$

Set $N := PQ$ and compute $\varphi_s(N) = \varphi_s(P)\varphi_s(Q)$. Pick a public exponent $E = e_1 + e_2 I$ such that

$$\gcd\big(e_1,\ \varphi(\mathrm{Re}(N))\big) = 1, \qquad \gcd\big(e_1 + e_2,\ \varphi(\mathrm{Tot}(N))\big) = 1.$$

Find $D = D_1 + D_2 I$ satisfying

$$D_1 \equiv e_1^{-1} \pmod{\varphi(\mathrm{Re}(N))}, \qquad D_1 + D_2 \equiv (e_1 + e_2)^{-1} \pmod{\varphi(\mathrm{Tot}(N))},$$

i.e. $D \equiv E^{-1} \pmod{\varphi_s(N)}$ under the congruence above. Publish (E, N) and keep D secret.

Encryption. Encode a message as $M = m + nI \in \mathbb{Z}(I)$ with

$$0 \le \mathrm{Re}(M) = m < \mathrm{Re}(N), \qquad 0 \le \mathrm{Tot}(M) = m + n < \mathrm{Tot}(N).$$

Compute the ciphertext

$$C \equiv M^E \pmod{N}.$$

Equivalently, under π_N,

$$\mathrm{Re}(C) \equiv m^{\,e_1} \pmod{\mathrm{Re}(N)}, \qquad \mathrm{Tot}(C) \equiv (m + n)^{\,e_1 + e_2} \pmod{\mathrm{Tot}(N)}.$$

Decryption. Recover M by

$$M \equiv C^D \pmod{N},$$

i.e.

$$\mathrm{Re}(M) \equiv \mathrm{Re}(C)^{\,D_1} \pmod{\mathrm{Re}(N)}, \qquad \mathrm{Tot}(M) \equiv \mathrm{Tot}(C)^{\,D_1 + D_2} \pmod{\mathrm{Tot}(N)}.$$

Example 5.6 (Neutrosophic RSA: key generation, encryption, decryption)**. Key generation.** Let $P = 5 + I$ and $Q = 7 + 4I$. Then $\gcd(5, 7) = 1$ and $\gcd(5 + 1,\ 7 + 4) = \gcd(6, 11) = 1$, so $\gcd(P, Q) = 1$ (componentwise). Compute

$$N = PQ = 35 + (5 \cdot 4 + 1 \cdot 7 + 1 \cdot 4)I = 35 + 31I, \quad \mathrm{Re}(N) = 35,\ \mathrm{Tot}(N) = 66.$$

Euler data:

$$\varphi(\mathrm{Re}(N)) = \varphi(35) = 24, \qquad \varphi(\mathrm{Tot}(N)) = \varphi(66) = 20.$$

Pick a public exponent $E = e_1 + e_2 I = 5 + 2I$; indeed $\gcd(5, 24) = 1$ and $\gcd(5 + 2, 20) = \gcd(7, 20) = 1$. Choose $D = D_1 + D_2 I$ with

$$D_1 \equiv 5^{-1} \pmod{24}\ (= 5), \qquad D_1 + D_2 \equiv 7^{-1} \pmod{20}\ (= 3).$$

Thus $D_2 \equiv 3 - 5 \equiv -2 \equiv 18 \pmod{20}$, and we take $D = 5 + 18I$. Publish (E, N), keep D secret.

Encryption. Encode a message $M = m + nI$ with $0 \le m < \mathrm{Re}(N) = 35$ and $0 \le m + n < \mathrm{Tot}(N) = 66$. Let $M = 12 + 9I$ (so $\mathrm{Re}(M) = 12$, $\mathrm{Tot}(M) = 21$). The ciphertext $C \equiv M^E \pmod{N}$ satisfies, via π_N,

$$\mathrm{Re}(C) \equiv 12^5 \pmod{35} \ = \ 17, \qquad \mathrm{Tot}(C) \equiv 21^7 \pmod{66} \ = \ 21,$$

(the second congruence uses $21^2 \equiv 45$, $21^3 \equiv 21$, hence $21^7 \equiv 21 \bmod 66$).

Decryption. Recover M by $M \equiv C^D \pmod{N}$, i.e.

$$\mathrm{Re}(M) \equiv 17^5 \pmod{35} = 12, \qquad \mathrm{Tot}(M) \equiv 21^{\,D_1 + D_2} = 21^{23} \equiv 21 \pmod{66}.$$

Thus the original $M = 12 + 9I$ is reconstructed (since $\mathrm{Re}(M) = 12$ and $\mathrm{Tot}(M) = 21$).

Theorem 5.7 (Correctness)**.** *Let (E, N) and D be as above, and let $C \equiv M^E \pmod{N}$. Then $C^D \equiv M \pmod{N}$.*

Proof. Apply π_N and use classical Euler's theorem componentwise. Since $D_1 \equiv e_1^{-1} \pmod{\varphi(\mathrm{Re}(N))}$ and $D_1 + D_2 \equiv (e_1 + e_2)^{-1} \pmod{\varphi(\mathrm{Tot}(N))}$, we have

$$\mathrm{Re}(C^D) \equiv \mathrm{Re}(M)^{\,e_1 D_1} \equiv \mathrm{Re}(M) \pmod{\mathrm{Re}(N)}, \qquad \mathrm{Tot}(C^D) \equiv \mathrm{Tot}(M)^{\,(e_1 + e_2)(D_1 + D_2)} \equiv \mathrm{Tot}(M) \pmod{\mathrm{Tot}(N)}.$$

Hence $\pi_N(C^D) = \pi_N(M)$, i.e. $C^D \equiv M \pmod{N}$. □

5.2 Neutrosophic ElGamal

ElGamal cryptosystem encrypts in a cyclic group: public key $y = g^x$; ciphertext $(g^k, m\, y^k)$; decryption uses secret x to recover m [227–230]. The neutrosophic ElGamal scheme attaches a triplet (T, I, F) to messages and propagates it through encryption, transmission, decryption, and decision aggregation.

Definition 5.8 (ElGamal Public–Key Cryptosystem)**.** [231–233] Let G be a cyclic group of prime order q with generator $g \in G$ (e.g. a subgroup of $\mathbb{F}_p^\times$ or an elliptic–curve group). The scheme consists of:

- **KeyGen:** Choose $x \leftarrow \mathbb{Z}_q$ uniformly and set $X := g^x \in G$. Public key pk $= (G, q, g, X)$; secret key sk $= x$.
- **Enc:** To encrypt $m \in G$ under pk, sample $r \leftarrow \mathbb{Z}_q$, set $R := g^r$ and the shared key $K := X^r = g^{xr}$. Output ciphertext $C = (R, S)$ with $S := m \cdot K$.
- **Dec:** Given $C = (R, S)$ and sk $= x$, compute $K := R^x$ and output $m := S \cdot K^{-1}$.

Proposition 5.9 (Correctness)**.** *For any $m \in G$, decryption returns m.*

Proof. Since $X = g^x$ and $R = g^r$, one has $K = X^r = g^{xr} = R^x$. Hence $S \cdot K^{-1} = (m \cdot K) \cdot K^{-1} = m$. □

Remark 5.10. IND-CPA security follows under the DDH (or CDH) assumption in G (proof omitted).

Definition 5.11 (Neutrosophic ElGamal over $\mathbb{Z}(I)$)**.** Let $\mathbb{Z}(I) = \{a + bI : a, b \in \mathbb{Z},\ I^2 = I\}$ be the neutrosophic integer ring. Fix a *neutrosophic modulus* $p = p_1 + p_2 I$ such that p_1 and $p_1 + p_2$ are primes, and work in the multiplicative group of units $(\mathbb{Z}(I)/\langle p\rangle)^\times$. Exponentiation in $\mathbb{Z}(I)$ is understood via the rule

$$(a + bI)^{c+dI} \;=\; a^c \;+\; I\Big((a + b)^{c+d} - a^c\Big),$$

and congruence mod p is the standard one induced by the canonical projection $\mathbb{Z}(I) \to \mathbb{Z}(I)/\langle p\rangle$.

- **Parameters:** Choose a generator $g = g_1 + g_2 I \in (\mathbb{Z}(I)/\langle p\rangle)^\times$.
- **KeyGen:** Pick secret $x = x_1 + x_2 I$ with $0 < x_1 < p_1 - 2$ and $0 < x_1 + x_2 < p_1 + p_2 - 2$. Set

  $$X \;:=\; g^{\,x} \pmod p.$$

  Public key pk $= (p, g, X)$; secret key sk $= x$.
- **Enc:** To encrypt $m \in (\mathbb{Z}(I)/\langle p\rangle)^\times$, sample $r = r_1 + r_2 I$ (with the same range conditions), and compute

  $$R := g^{\,r} \pmod p, \qquad K := X^{\,r} \pmod p, \qquad S := m \cdot K \pmod p.$$

  Output $C = (R, S)$.
- **Dec:** Given $C = (R, S)$ and sk $= x$, compute $K := R^{\,x} \pmod p$ and output $m := S \cdot K^{-1} \pmod p$ (multiplicative inverse taken in $(\mathbb{Z}(I)/\langle p\rangle)^\times$).

Example 5.12 (Neutrosophic ElGamal over $\mathbb{Z}(I)$: small worked instance I)**.** Let $p = 5 + 2I$, so $p_1 = 5$ and $p_1 + p_2 = 7$ are primes. Work in the unit group $(\mathbb{Z}(I)/\langle p\rangle)^\times \cong \mathbb{Z}_5^\times \times \mathbb{Z}_7^\times$, via the canonical projection

$$\pi_p :\ a + bI \;\longmapsto\; \big(a \bmod 5,\ a + b \bmod 7\big).$$

Choose the base ("generator") $g = 2 + I$, for which

$$\pi_p(g) = (2,\ 3).$$

(In $\mathbb{Z}_5^\times$, 2 has order 4; in $\mathbb{Z}_7^\times$, 3 has order 6; hence g generates a subgroup of order $\mathrm{lcm}(4, 6) = 12$.)

KeyGen. Pick secret $x = 2 + 2I$ (so $0 < x_1 < 5 - 2$ and $0 < x_1 + x_2 < 7 - 2$ hold). Using the neutrosophic power rule $(a + bI)^{c+dI} = a^c + I\big((a + b)^{c+d} - a^c\big)$, one gets componentwise

$$\pi_p\big(g^x\big) = \big(2^{\,2} \bmod 5,\ 3^{\,2+2} \bmod 7\big) = (4,\ 4).$$

A representative is $X = 4 + 0 \cdot I$. The public key is (p, g, X), and the secret key is x.

Message. Let $m = 3 + I$; then $\pi_p(m) = (3,\ 4)$, which is a unit pair.

Enc. Choose ephemeral $r = 1 + 3I$ (valid: $0 < 1 < 3, 0 < 1 + 3 < 5$). Then

$$\pi_p(R) = \pi_p(g^r) = \left(2^1,\ 3^{1+3}\right) = (2,\ 3^4) = (2,\ 4).$$

A representative is $R = 2 + 2I$ (since $2 \bmod 5 = 2$ and $2 + 2 \bmod 7 = 4$). Also

$$\pi_p(K) = \pi_p(X^r) = \left(4^1,\ 4^{1+3}\right) = (4,\ 4),$$

so we may take $K = 4 + 0 \cdot I$. The ciphertext second component is

$$S \equiv m \cdot K \pmod{p} \implies \pi_p(S) = \left(3 \cdot 4 \bmod 5,\ 4 \cdot 4 \bmod 7\right) = (2,\ 2),$$

for which a representative is $S = 2 + 0 \cdot I$. Hence $C = (R, S) = (2 + 2I,\ 2)$.

Dec. Compute $K = R^x$:

$$\pi_p(K) = \pi_p\big((2,4)^{(2,\ 4)}\big) = \left(2^2 \bmod 5,\ 4^4 \bmod 7\right) = (4,\ 4),$$

so K^{-1} corresponds to the pair $(4^{-1} \bmod 5,\ 4^{-1} \bmod 7) = (4,\ 2)$, e.g. the element $4 + 5I$. Recover

$$\pi_p(m) = \pi_p(S) \cdot \pi_p(K^{-1}) = (2,\ 2) \cdot (4,\ 2) = (8 \bmod 5,\ 4 \bmod 7) = (3,\ 4),$$

which indeed lifts to $m = 3 + I$.

Example 5.13 (Neutrosophic ElGamal over $\mathbb{Z}(I)$: small worked instance II)**.** Let $p = 11 + 2I$, so $p_1 = 11$ and $p_1 + p_2 = 13$ are primes. Use

$$\pi_p :\ a + bI \longmapsto \left(a \bmod 11,\ a + b \bmod 13\right), \quad (\mathbb{Z}(I)/\langle p\rangle)^\times \cong \mathbb{Z}_{11}^\times \times \mathbb{Z}_{13}^\times.$$

Choose $g = 2 + 0 \cdot I$ so that $\pi_p(g) = (2,\ 2)$; 2 is a primitive root mod 11 (order 10) and mod 13 (order 12).

KeyGen. Take secret $x = 7 + 3I$ (valid since $0 < 7 < 9$ and $0 < 7 + 3 = 10 < 11$). Then

$$\pi_p(X) = \pi_p(g^x) = \left(2^7 \bmod 11,\ 2^{10} \bmod 13\right) = (7,\ 10),$$

which admits the representative $X = 7 + 3I$ (since $7 \bmod 11 = 7$, $7 + 3 \bmod 13 = 10$). Public key (p, g, X), secret key x.

Message. Let $m = 9 + 5I$; then $\pi_p(m) = (9,\ 1)$ is a unit pair.

Enc. Choose $r = 4 + 5I$ (valid: $0 < 4 < 9, 0 < 4 + 5 = 9 < 11$). Then

$$\pi_p(R) = \pi_p(g^r) = \left(2^4,\ 2^9\right) = (5,\ 5),$$

so we may take $R = 5$. Also

$$\pi_p(K) = \pi_p(X^r) = \left(7^4 \bmod 11,\ 10^9 \bmod 13\right) = (3,\ 12),$$

e.g. $K = 3 + 9I$. Compute $S \equiv m \cdot K \pmod{p}$. In $\mathbb{Z}(I)$,

$$(9 + 5I)(3 + 9I) = 27 + (9 \cdot 9 + 5 \cdot 3 + 5 \cdot 9)I = 27 + 141I.$$

Reduce via π_p:

$$\pi_p(S) = \left(27 \bmod 11,\ 27 + 141 \bmod 13\right) = (5,\ 12),$$

for which a representative is $S = 5 + 7I$. Hence $C = (R, S) = (5,\ 5 + 7I)$.

Dec. Compute $K = R^x$:

$$\pi_p(K) = \pi_p\big((5,5)^{(7,\ 10)}\big) = \left(5^7 \bmod 11,\ 5^{10} \bmod 13\right) = (3,\ 12).$$

Thus K^{-1} has pair $(3^{-1} \bmod 11,\ 12^{-1} \bmod 13) = (4,\ 12)$; one representative is $4 + 8I$. Recover

$$\pi_p(m) = \pi_p(S) \cdot \pi_p(K^{-1}) = (5,\ 12) \cdot (4,\ 12) = (20 \bmod 11,\ 144 \bmod 13) = (9,\ 1),$$

which lifts back to $m = 9 + 5I$ as desired.

Lemma 5.14 (Correctness in $\mathbb{Z}(I)/\langle p\rangle$)**.** *Decryption recovers the plaintext m.*

Proof. By construction $X = g^x$ and $R = g^r$ in $(\mathbb{Z}(I)/\langle p\rangle)^\times$. Hence $K = X^r = (g^x)^r = g^{xr} = (g^r)^x = R^x$, so $S \cdot K^{-1} = (m \cdot K) \cdot K^{-1} = m$. □

5.3 Neutrosophic AES

The Advanced Encryption Standard (AES) is a symmetric block cipher using 128-bit blocks and 128/192/256-bit keys, standardized worldwide for confidentiality [234–237]. A Neutrosophic AES scheme augments AES with triplet metadata measuring truth, indeterminacy, and falsity of security assumptions and implementation contexts.

Definition 5.15 (Advanced Encryption Standard (AES))**.** Let $\mathbb{F}_2$ be the field of two elements and let

$$\mathbb{F}_{2^8} \;\cong\; \mathbb{F}_2[z]/\langle m(z)\rangle, \quad m(z) = z^8 + z^4 + z^3 + z + 1,$$

be the byte field. Represent a 128-bit *state* as a 4×4 byte matrix $S = (s_{i,j})_{0\le i,j\le 3}$ over $\mathbb{F}_{2^8}$ (column-major); represent the secret key K as $N_k \in \{4, 6, 8\}$ 32-bit words (key sizes 128/192/256 bits). Let $N_r \in \{10, 12, 14\}$ be the corresponding number of rounds.

Round components.

1. **SubBytes** (S-box): For each byte $a \in \mathbb{F}_{2^8}$, define

   $$\mathrm{S}(a) = A(a^{-1}) + b,$$

   where a^{-1} is multiplicative inverse in $\mathbb{F}_{2^8}$ with $0^{-1} := 0$, $A : \mathbb{F}_{2^8} \to \mathbb{F}_{2^8}$ is a fixed $\mathbb{F}_2$-linear automorphism (affine over $\mathbb{F}_2^8$), and $b \in \mathbb{F}_{2^8}$ is a fixed byte. Apply S entrywise to S.

2. **ShiftRows**: For each row $i \in \{0, 1, 2, 3\}$, rotate the row left by i bytes:

   $$(s_{i,0}, s_{i,1}, s_{i,2}, s_{i,3}) \mapsto (s_{i,i}, s_{i,i+1}, s_{i,i+2}, s_{i,i+3}),$$

   with column indices modulo 4.

3. **MixColumns**: Multiply each column $c = (s_{0,j}, s_{1,j}, s_{2,j}, s_{3,j})^\top$ by the fixed MDS matrix M over $\mathbb{F}_{2^8}$,

   $$M = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 1 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix}, \quad c \mapsto M\,c,$$

   where coefficients are field elements (hex) and multiplication/addition are in $\mathbb{F}_{2^8}$.

4. **AddRoundKey**: Bitwise XOR the round key $K^{(r)}$ into the state: $S \mapsto S \oplus K^{(r)}$ (XOR is addition in $\mathbb{F}_2^{128}$).

Key schedule. From the key words $W[0], \ldots, W[N_k - 1]$, generate words $W[i]$ for $i = N_k, \ldots, 4(N_r + 1) - 1$ by

$$W[i] = \begin{cases} W[i - N_k] \;\oplus\; \mathrm{SubWord}(\mathrm{RotWord}(W[i - 1])) \;\oplus\; \mathrm{Rcon}[i/N_k], & i \equiv 0 \pmod{N_k}, \\ W[i - N_k] \;\oplus\; \mathrm{SubWord}(W[i - 1]), & N_k = 8 \text{ and } i \equiv 4 \pmod 8, \\ W[i - N_k] \;\oplus\; W[i - 1], & \text{otherwise}, \end{cases}$$

where RotWord rotates a 32-bit word by 8 bits, SubWord applies S to each byte, and $\mathrm{Rcon}[\cdot]$ is the standard round-constant sequence over $\mathbb{F}_{2^8}$. The round keys $K^{(r)}$ are formed from $W[4r], \ldots, W[4r + 3]$.

Encryption. Given plaintext state $S^{(0)}$ and round keys $K^{(0)}, \ldots, K^{(N_r)}$:

$$\begin{aligned} &\text{Initial AddRoundKey: } S \leftarrow S^{(0)} \oplus K^{(0)}; \\ &\text{For } r = 1, \ldots, N_r - 1: \; S \leftarrow \mathrm{MixColumns} \circ \mathrm{ShiftRows} \circ \mathrm{SubBytes}(S) \;\oplus\; K^{(r)}; \\ &\text{Final round: } S \leftarrow \mathrm{ShiftRows} \circ \mathrm{SubBytes}(S) \;\oplus\; K^{(N_r)}. \end{aligned}$$

The output state is the ciphertext C; decryption uses inverse transforms S^{-1}, inverse row shifts, and inverse MixColumns.

Definition 5.16 (Neutrosophic Advanced Encryption Standard (Neutrosophic AES))**.** Fix a t-norm $t : [0,1]^2 \to [0,1]$ and its dual t-conorm $s : [0,1]^2 \to [0,1]$, both associative, commutative, and monotone. A *neutrosophic label* is a triplet $v = (T, I, F) \in [0,1]^3$ representing degrees of truth/assuredness (T), indeterminacy/uncertainty (I), and falsity/risk (F). Let $\rho \in [0,1]$ be a *reliability parameter* capturing implementation, side-channel, and operational conditions for the encryption pipeline.

Given a plaintext block $P \in \{0,1\}^{128}$ with label $v_P = (T_P, I_P, F_P)$ and a key K with label $v_K = (T_K, I_K, F_K)$, the *neutrosophic AES transform* is the map

$$\mathrm{NAES}_{K,\rho} : \; (P, v_P) \;\longmapsto\; \big(E_K(P),\; \Phi_\rho(v_P, v_K)\big),$$

where E_K is the classical AES encryption and the label mixer Φ_ρ is

$$\Phi_\rho(v_P, v_K) = \Big(T^+,\ I^+,\ F^+ \Big)$$

with

$$T^+ \ = \ t\big(t(T_P, T_K),\ \rho\big), \qquad I^+ \ = \ s\big(s(I_P, I_K),\ 1-\rho\big), \qquad F^+ \ = \ s\big(s(F_P, F_K),\ 1-\rho\big).$$

Thus T^+ increases with plaintext/key assurance and reliability; I^+ aggregates indeterminacies and grows when reliability decreases; F^+ collects falsity/risk and also inflates when reliability is low. The decryption layer $\mathrm{NAES}^{-1}_{K,\rho}$ is defined analogously by replacing E_K with D_K and using the same Φ_ρ to propagate labels backward (or a calibrated inverse if desired).

Example 5.17 (Neutrosophic AES: high–reliability data–center encryption)**.** We instantiate the neutrosophic label mixer with the product t–norm $t(x, y) = xy$ and the probabilistic t–conorm $s(x, y) = x + y - xy$. Let the plaintext block $P \in \{0, 1\}^{128}$ carry label $v_P = (T_P, I_P, F_P) = (0.98,\ 0.01,\ 0.01)$, the key K carry $v_K = (0.97,\ 0.02,\ 0.01)$, and the reliability parameter be $\rho = 0.95$ (well–hardened implementation).

The neutrosophic AES transform outputs ciphertext $C = E_K(P)$ and label $\Phi_\rho(v_P, v_K) = (T^+, I^+, F^+)$ with

$$\begin{aligned}
T^+ &= t\big(t(T_P, T_K),\ \rho\big) = (0.98 \cdot 0.97) \cdot 0.95 = 0.9506 \cdot 0.95 \approx 0.903,\\
I^+ &= s\big(s(I_P, I_K),\ 1-\rho\big) = s\big(0.01 + 0.02 - 0.01 \cdot 0.02,\ 0.05\big)\\
&= s(0.0298,\ 0.05) = 0.0298 + 0.05 - 0.0298 \cdot 0.05 \approx 0.0783,\\
F^+ &= s\big(s(F_P, F_K),\ 1-\rho\big) = s\big(0.01 + 0.01 - 0.01 \cdot 0.01,\ 0.05\big)\\
&= s(0.0199,\ 0.05) = 0.0199 + 0.05 - 0.0199 \cdot 0.05 \approx 0.0689.
\end{aligned}$$

Thus the output label ($\approx 0.903,\ 0.078,\ 0.069$) reflects high assuredness, with small indeterminacy and risk contributed by residual uncertainty and the (small) unreliability $1 - \rho$.

Example 5.18 (Neutrosophic AES: low–reliability edge/IoT node)**.** Use the same mixers $t(x, y) = xy$ and $s(x, y) = x+y-xy$. Let a sensor node encrypt P with label $v_P = (0.80,\ 0.15,\ 0.05)$ using a key with label $v_K = (0.60,\ 0.25,\ 0.15)$. Due to side–channel exposure and weak entropy, take $\rho = 0.40$.

Then $C = E_K(P)$ and

$$\begin{aligned}
T^+ &= (0.80 \cdot 0.60) \cdot 0.40 = 0.48 \cdot 0.40 = 0.192,\\
I^+ &= s\big(s(0.15, 0.25),\ 0.60\big) = s\big(0.15 + 0.25 - 0.0375,\ 0.60\big) = s(0.3625,\ 0.60)\\
&= 0.3625 + 0.60 - 0.3625 \cdot 0.60 = 0.9625 - 0.2175 = 0.745,\\
F^+ &= s\big(s(0.05, 0.15),\ 0.60\big) = s\big(0.05 + 0.15 - 0.0075,\ 0.60\big) = s(0.1925,\ 0.60)\\
&= 0.1925 + 0.60 - 0.1925 \cdot 0.60 = 0.7925 - 0.1155 = 0.677.
\end{aligned}$$

The output label $(0.192,\ 0.745,\ 0.677)$ indicates low assuredness and large indeterminacy/risk, consistent with poor reliability and imperfect key/plaintext provenance in a hostile environment.

Theorem 5.19 (Reduction to classical AES)**.** *If $v_P = (1, 0, 0)$, $v_K = (1, 0, 0)$, and $\rho = 1$, then*

$$\mathrm{NAES}_{K,\rho}(P, v_P) = \big(E_K(P),\ (1, 0, 0)\big),$$

so the neutrosophic AES reduces to the classical AES on data and carries a "certain" label. More generally, for any P, when the labels are $(1, 0, 0)$ and $\rho = 1$, the neutrosophic layer leaves the ciphertext unchanged and the label equal to $(1, 0, 0)$.

Proof. With t associative and $t(1, x) = x$, we have $T^+ = t(t(1, 1), 1) = 1$. With s associative, $s(0, x) = x$, and $1 - \rho = 0$, we obtain $I^+ = s(s(0, 0), 0) = 0$ and $F^+ = s(s(0, 0), 0) = 0$. Hence the label is $(1, 0, 0)$. The data path is exactly $E_K(P)$ by definition. □

Theorem 5.20 (Neutrosophic structure on the ciphertext space)**.** *Let $C = \{0, 1\}^{128}$ be the AES ciphertext space. The map*

$$\Lambda_{K,\rho} : \ C \ \longrightarrow \ [0, 1]^3, \qquad \Lambda_{K,\rho}\big(E_K(P)\big) \ := \ \Phi_\rho(v_P, v_K),$$

endows C with a neutrosophic-set structure: each ciphertext is annotated by a triplet $(T^+, I^+, F^+) \in [0, 1]^3$ produced by monotone t/s-based aggregation of source/key labels and reliability. Consequently, $(C, \Lambda_{K,\rho})$ is a neutrosophic set in the sense of assigning to each element degrees of truth, indeterminacy, and falsity in $[0, 1]$.

Proof. By construction, $\Phi_\rho : [0, 1]^3 \times [0, 1]^3 \to [0, 1]^3$ is built from t and s, which map $[0, 1]^2$ to $[0, 1]$ and are monotone and bounded. Hence $T^+, I^+, F^+ \in [0, 1]$ for all inputs, and the assignment is well-defined on C. This is exactly the data of a neutrosophic set on C. □

5.4 Cryptographic Neutrosophic Hash Function

A cryptographic hash function maps arbitrary-length inputs to fixed-size digests, enabling preimage, second-preimage, and collision resistance against computationally efficient adversaries [238–241]. A neutrosophic hash function pairs a classical hash with triplet labels quantifying truth, indeterminacy, and falsity of security under uncertainty.

Definition 5.21 (Cryptographic Hash Function)**.** [242] Let $\{0,1\}^*$ denote the set of all finite binary strings and let $n \in \mathbb{N}$. A (deterministic) *hash function* with n-bit output is a map

$$H : \{0,1\}^* \longrightarrow \{0,1\}^n,$$

that is efficiently computable (polynomial time in the input length) and satisfies the standard hardness properties against any probabilistic polynomial–time (PPT) adversary $\mathcal{A}$:

1. **Preimage resistance.** For $y \leftarrow \{0,1\}^n$ uniform, the advantage
$$\mathrm{Adv}^{\mathrm{pre}}_H(\mathcal{A}) := \Pr\left[\, x \leftarrow \mathcal{A}(1^\lambda, y) \ \text{ s.t. } \ H(x) = y \,\right]$$
is negligible in the security parameter λ.
2. **Second–preimage resistance.** For $x \leftarrow \{0,1\}^*$ (drawn by the experiment),
$$\mathrm{Adv}^{\mathrm{2pre}}_H(\mathcal{A}) := \Pr\left[\, x' \leftarrow \mathcal{A}(1^\lambda, x) \ \text{ s.t. } \ x' \neq x,\ H(x') = H(x) \,\right]$$
is negligible.
3. **Collision resistance.** The advantage
$$\mathrm{Adv}^{\mathrm{coll}}_H(\mathcal{A}) := \Pr\left[\, (x, x') \leftarrow \mathcal{A}(1^\lambda) \ \text{ s.t. } \ x \neq x',\ H(x) = H(x') \,\right]$$
is negligible.

Here "negligible" means smaller than $1/p(\lambda)$ for every polynomial p and all sufficiently large λ.

Definition 5.22 (Neutrosophic Hash Function)**.** Fix a t-norm $t : [0,1]^2 \to [0,1]$ and its dual t-conorm $s : [0,1]^2 \to [0,1]$ (both associative, commutative, and monotone). A *neutrosophic label* is a triplet $(T, I, F) \in [0,1]^3$ interpreted as degrees of truth/assuredness (T), indeterminacy/uncertainty (I), and falsity/risk (F).

A *neutrosophic hash function* is a pair

$$\mathsf{NH} = (H,\ \mathsf{Prof}),$$

where $H : \{0,1\}^* \to \{0,1\}^n$ is a classical hash function and

$$\mathsf{Prof} : \{\mathrm{pre}, \mathrm{2pre}, \mathrm{coll}\} \times \mathcal{A} \times \mathbb{N} \longrightarrow [0,1]^3$$

is a *neutrosophic security profile* that, for each property $\pi \in \{\mathrm{pre}, \mathrm{2pre}, \mathrm{coll}\}$, adversary $\mathcal{A}$, and security parameter λ, returns a triplet

$$\mathsf{Prof}(\pi, \mathcal{A}, \lambda) = \big(T_\pi(\mathcal{A}, \lambda),\ I_\pi(\mathcal{A}, \lambda),\ F_\pi(\mathcal{A}, \lambda)\big),$$

subject to the following calibration with the usual security *advantages*:

$$\mathrm{Adv}^{\pi}_H(\mathcal{A}; \lambda) \in [0,1], \quad u_\pi(\mathcal{A}, \lambda) \in [0,1] \text{ (an uncertainty bound)},$$

and the aggregation

$$T_\pi = (1 - u_\pi)\,\big(1 - \mathrm{Adv}^{\pi}_H\big), \qquad I_\pi = u_\pi, \qquad F_\pi = (1 - u_\pi)\,\mathrm{Adv}^{\pi}_H.$$

Hence $T_\pi + I_\pi + F_\pi = 1$, with T_π grading achieved resistance, F_π grading observed/estimated vulnerability, and I_π capturing modeling, measurement, or operational uncertainty (e.g., finite sampling, side–channel variability). The mapping Prof is *monotone* in the sense that larger advantages (or larger uncertainties) do not decrease F_π (or I_π), and do not increase T_π.

Optionally, to annotate individual digests, one may equip NH with a *digest labeler*

$$\Lambda : \{0,1\}^* \longrightarrow [0,1]^3, \qquad x \longmapsto \Lambda(x) = (T_x, I_x, F_x),$$

where Λ quantifies, for the specific input x, the stability of $H(x)$ under a prescribed perturbation model and environment; e.g.,

$$T_x = (1 - u_x)\,\Pr\big[\, H(X') = H(x) \,\big], \quad I_x = u_x, \quad F_x = (1 - u_x)\,\big(1 - \Pr[H(X') = H(x)]\big),$$

with X' drawn from a task–dependent neighborhood of x and $u_x \in [0,1]$ an uncertainty mass.

Example 5.23 (Secure Boot with Environmental Variability (Digest–Level Neutrosophic Labeling))**.** Consider an IoT device that verifies firmware integrity by hashing the image $x \in \{0,1\}^*$ and checking $H(x)$ at boot. During field operation, temperature and supply noise induce bit upsets; model a perturbed image X' by independent flips with probability $p \in [0, 1/2)$ on each bit of x. Let

$$p_{\text{stab}} \;:=\; \Pr\big[\, H(X') = H(x) \,\big]$$

be the *digest stability* under this perturbation. Estimate p_{stab} by M Monte Carlo trials: $p_{\text{stab}} \approx \hat{p} := \frac{1}{M}\sum_{m=1}^{M} \mathbf{1}\{H(x^{(m)}) = H(x)\}$. Let $u_x \in [0,1]$ encode the empirical uncertainty, e.g. from a two–sided $(1-\alpha)$ Clopper–Pearson interval half–width w_α via $u_x := \min\{1,\ w_\alpha/c\}$ with a scaling constant $c > 0$.

Attach to x the neutrosophic digest label (cf. the definition of Λ):

$$T_x \;:=\; (1-u_x)\,\hat{p}, \qquad I_x \;:=\; u_x, \qquad F_x \;:=\; (1-u_x)\,(1-\hat{p}).$$

Here T_x grades the likelihood that environmental noise does *not* break integrity checks (hash unchanged), F_x grades *vulnerability* to bit–flip–induced digest changes (false boot failures), and I_x captures finite–sample and modeling uncertainty. Aggregating over multiple firmware segments $\{x_j\}$ can be done with a t–norm/t–conorm mixer.

Example 5.24 (Post–Quantum Readiness Audit (Profile–Level Neutrosophic Security))**.** Let $H : \{0,1\}^* \to \{0,1\}^n$ be slated for long–term use in a blockchain application. We assess resistance to (a) preimage, (b) second–preimage, and (c) collision attacks in a *post–quantum* threat model. Suppose an adversary $\mathcal{A}$ makes Q quantum oracle queries. Using standard bounds, take

$$\mathrm{Adv}^{\mathrm{pre}}_H(\mathcal{A}) \;\approx\; \min\Big\{1,\ \frac{Q}{2^{n/2}}\Big\}, \qquad \mathrm{Adv}^{\mathrm{coll}}_H(\mathcal{A}) \;\approx\; \min\Big\{1,\ \frac{Q}{2^{n/3}}\Big\},$$

and set $\mathrm{Adv}^{\mathrm{2pre}}_H(\mathcal{A}) \approx \mathrm{Adv}^{\mathrm{pre}}_H(\mathcal{A})$ for a conservative audit. Let $u_\pi \in [0,1]$ quantify model uncertainty (e.g. oracle idealization, side–channels, key–schedule leakage) for property $\pi \in \{\mathrm{pre}, \mathrm{2pre}, \mathrm{coll}\}$.

Define the neutrosophic security profile $\mathsf{Prof}(\pi, \mathcal{A}, \lambda) = (T_\pi, I_\pi, F_\pi)$ by

$$T_\pi \;:=\; (1-u_\pi)\,\big(1 - \mathrm{Adv}^{\pi}_H(\mathcal{A})\big), \quad I_\pi \;:=\; u_\pi, \quad F_\pi \;:=\; (1-u_\pi)\,\mathrm{Adv}^{\pi}_H(\mathcal{A}).$$

Thus, if $n = 256$ and $Q = 2^{64}$ queries are feasible, then $\mathrm{Adv}^{\mathrm{pre}} \approx 2^{64}/2^{128} = 2^{-64}$ (very small) yields $T_{\mathrm{pre}} \approx (1-u_{\mathrm{pre}})(1 - 2^{-64}) \approx 1 - u_{\mathrm{pre}}$, while a more pessimistic collision exponent (e.g. multi–target structure) could raise $\mathrm{Adv}^{\mathrm{coll}}$ and hence F_{coll}. The triplets (T_π, I_π, F_π) therefore provide a *graded*, uncertainty–aware dashboard of cryptanalytic posture for operational risk governance and migration planning.

Theorem 5.25 (Reduction to the classical notion)**.** *If, for all PPT adversaries $\mathcal{A}$ and all sufficiently large λ, the classical advantages $\mathrm{Adv}^{\pi}_H(\mathcal{A};\lambda)$ are negligible and the uncertainties $u_\pi(\mathcal{A},\lambda) = 0$, then the neutrosophic profile satisfies*

$$T_\pi(\mathcal{A},\lambda) = 1 - \mathrm{Adv}^{\pi}_H(\mathcal{A};\lambda) \to 1, \quad I_\pi(\mathcal{A},\lambda) = 0, \quad F_\pi(\mathcal{A},\lambda) = \mathrm{Adv}^{\pi}_H(\mathcal{A};\lambda) \to 0,$$

so NH *collapses to the standard (classical) hash security guarantees.*

Proof. Immediate from the defining equations for (T_π, I_π, F_π) with $u_\pi = 0$ and Adv^{π}_H negligible. □

5.5 Neutrosophic IT Service Management

We present the definition of the Neutrosophic IT Service Management Framework, along with a concrete example.

Definition 5.26 (Mathematical Framework for Neutrosophic IT Service Management)**.** The *Neutrosophic IT Service Management* (NITSM) framework extends the conventional (and fuzzy) IT Service Management framework by incorporating neutrosophic uncertainty into key evaluation functions. It is modeled as a tuple

$$\mathcal{NITSM} = \big(S, I, P, M, \tilde{Q}_N, \tilde{U}_N, \tilde{R}_N, C, \varphi, \psi\big),$$

where:

- S is a finite set of IT services (e.g., online banking, email, cloud storage).
- I is a finite set of IT infrastructure components (such as servers, databases, network devices) with attributes like reliability $r(i)$ and operational cost $c(i)$.
- P is a finite set of IT management processes (e.g., Incident Management, Change Management) with a compliance function $\psi : P \times S \to \{0,1\}$.

- $M : S \to 2^I \times 2^P$ is an allocation mapping that assigns to each service $s \in S$ a subset of infrastructure $I_s \subseteq I$ and processes $P_s \subseteq P$.
- C represents cost parameters, which may include overall budget constraints.
- $\varphi : 2^I \to [0, 1]$ is an aggregation function that, for example, combines the reliabilities of the infrastructure components assigned to a service.
- $\tilde{Q}_N : S \to NS(\mathbb{R}^+)$ is a *neutrosophic quality function* that assigns each service a neutrosophic evaluation of quality (e.g., a set of values with corresponding degrees of truth, indeterminacy, and falsity).
- $\tilde{U}_N : S \to NS(\mathbb{R})$ is a *neutrosophic utility function* capturing economic evaluations (such as revenue and cost) in a neutrosophic way.
- $\tilde{R}_N : S \to NS([0, 1])$ is a *neutrosophic reliability function* assigning to each service a neutrosophic measure of overall reliability.

In this framework, the neutrosophic functions $\tilde{Q}_N$, $\tilde{U}_N$, and $\tilde{R}_N$ map each service to a neutrosophic number, that is, a triple (T, I, F) (or a set thereof) capturing degrees of truth, indeterminacy, and falsity. Thus, the NITSM framework naturally reflects not only the inherent uncertainty but also the indeterminacy and conflicting evidence present in real IT service management contexts.

5.6 Neutrosophic Service Integration and Management

Neutrosophic Service Integration and Management generalizes SIAM by representing quality, cost, integration with neutrosophic triples capturing truth, indeterminacy, and falsity. The definitions and related concepts of Neutrosophic Service Integration and Management are provided below.

Definition 5.27 (Mathematical Framework for Neutrosophic Service Integration and Management)**.** To incorporate uncertainty more richly, we extend the above framework to the neutrosophic domain. In the *Neutrosophic Service Integration and Management* (NSIM) framework, uncertainty is modeled using neutrosophic numbers—that is, triples capturing degrees of truth, indeterminacy, and falsity.

Specifically, we define:

- The sets S, P, and the mapping $M : S \to \mathcal{P}(P) \setminus \{\emptyset\}$ remain as in the conventional framework.
- For each service $s \in S$ and each provider $p \in M(s)$, a *neutrosophic quality* is assigned:
$$\tilde{q}_N(p, s) \in NS([0, 1]),$$
where a neutrosophic number is a triple
$$\tilde{q}_N(p, s) = \big(T_q(p, s),\ I_q(p, s),\ F_q(p, s)\big),$$
with $T_q(p, s), I_q(p, s), F_q(p, s) \in [0, 1]$ and
$$0 \le T_q(p, s) + I_q(p, s) + F_q(p, s) \le 3.$$
- Similarly, for cost we define a *neutrosophic cost*:
$$\tilde{c}_N(p, s) \in NS(\mathbb{R}^+),$$
and for the integration overhead,
$$\tilde{\xi}_N(s) \in NS(\mathbb{R}^+).$$
- Let $\tilde{\varphi}_N$ be a neutrosophic aggregation operator that combines a set of neutrosophic quality numbers into a single neutrosophic quality measure.

Thus, the NSIM framework is modeled as the tuple
$$\mathcal{NSIM} = \Big(S,\ P,\ M,\ \{\tilde{q}_N(p, s)\}_{p \in M(s),\, s \in S},\ \{\tilde{c}_N(p, s)\}_{p \in M(s),\, s \in S},\ \tilde{\varphi}_N,\ \tilde{\xi}_N\Big).$$
For each service $s \in S$, we define the overall neutrosophic quality and cost as:
$$\tilde{Q}_N(s) = \tilde{\varphi}_N\Big(\{\tilde{q}_N(p, s) : p \in M(s)\}\Big),$$
$$\tilde{C}_N(s) = \bigoplus_{p \in M(s)} \tilde{c}_N(p, s) \oplus \tilde{\xi}_N(s),$$
where $\bigoplus$ denotes the neutrosophic addition operation on neutrosophic numbers.

This framework generalizes conventional Service Integration and Management by representing quality, cost, and overhead as neutrosophic numbers—thereby explicitly modeling uncertainty (truth), indeterminacy, and falsity.

5.7 Neutrosophic ITIL 4 Framework

The Neutrosophic ITIL 4 Framework encodes all components—principles, constraints, practices, transformation, and improvement—as neutrosophic triples of truth, indeterminacy, and falsity. The definitions and related concepts of Mathematical Framework for Neutrosophic ITIL 4 Framework are provided below.

Definition 5.28 (Neutrosophic ITIL 4 Framework)**.** The *Neutrosophic ITIL 4 Framework* generalizes the Fuzzy ITIL 4 Framework by endowing every component with a neutrosophic structure. It is defined as the quintuple

$$\widetilde{NITIL4} = \left(\widetilde{\mathcal{G}}_N,\ \widetilde{\mathcal{C}}_N,\ \widetilde{\mathcal{T}}_N,\ \widetilde{\mathcal{P}}_N,\ \widetilde{\mathcal{I}}_N\right),$$

with the following components:

1. **Neutrosophic Guiding Principles:** Let $\mathcal{U}_G$ be the universe of candidate guiding principles. Then the neutrosophic set of guiding principles is defined by a triple membership function

   $$\eta_{\widetilde{\mathcal{G}}} : \mathcal{U}_G \to [0,1]^3,$$

   where for each $g \in \mathcal{U}_G$,

   $$\eta_{\widetilde{\mathcal{G}}}(g) = \big(T_{\widetilde{\mathcal{G}}}(g),\ I_{\widetilde{\mathcal{G}}}(g),\ F_{\widetilde{\mathcal{G}}}(g)\big).$$

2. **Neutrosophic Governance Constraints:** Suppose a quality metric $q \in \mathbb{R}$ is used to assess a constraint. Its neutrosophic satisfaction is given by

   $$\eta_{\widetilde{\mathcal{C}}}(q) = \big(T_{\widetilde{\mathcal{C}}}(q),\ I_{\widetilde{\mathcal{C}}}(q),\ F_{\widetilde{\mathcal{C}}}(q)\big),$$

   where—for example—we may specify

   $$T_{\widetilde{\mathcal{C}}}(q) = \begin{cases} 0, & q < 70, \\ \frac{q-70}{10}, & 70 \le q \le 80, \\ 1, & q > 80, \end{cases}$$

   and assign suitable indeterminacy $I_{\widetilde{\mathcal{C}}}(q)$ (say, a small constant when q is near the threshold) and falsity $F_{\widetilde{\mathcal{C}}}(q)$ such that

   $$T_{\widetilde{\mathcal{C}}}(q) + I_{\widetilde{\mathcal{C}}}(q) + F_{\widetilde{\mathcal{C}}}(q) = 1 \quad \text{or is otherwise appropriately bounded.}$$

3. **Neutrosophic Service Value Chain Transformation:** Extend the crisp transformation $T : X \to Y$ to a neutrosophic mapping. For each $x \in X$, instead of a single output $T(x)$, we associate the neutrosophic value

   $$\eta_{\widetilde{T}}(x) = \left(T_{\widetilde{T}}(x),\ I_{\widetilde{T}}(x),\ F_{\widetilde{T}}(x)\right) \in [0,1]^3.$$

   In many practical cases one may define

   $$\widetilde{T}(x) = T(x) \quad \text{with confidence } \eta_{\widetilde{T}}(x).$$

4. **Neutrosophic Practices:** Let

   $$\widetilde{\mathcal{P}}_N = \{\widetilde{p}_1,\ \widetilde{p}_2, \ldots,\ \widetilde{p}_k\},$$

   where each neutrosophic practice $\widetilde{p}_i : Z_i \to W_i$ is paired with a triple membership function

   $$\eta_{\widetilde{p}_i} : W_i \to [0,1]^3.$$

   Their sequential composition, following neutrosophic aggregation rules, produces the overall transformation $\widetilde{T}_N$.

5. **Neutrosophic Continual Improvement:** Extend the improvement function $I : \mathbb{N}_0 \to \mathbb{R}$ to

   $$\widetilde{I}_N : \mathbb{N}_0 \to \mathbb{R},$$

   along with an associated neutrosophic confidence triple (often taken as $(1,0,0)$ for full certainty).

Finally, the neutrosophic service value function is given by

$$\eta_{\widetilde{v}} : Y \to [0,1]^3,$$

assigning to each output y a triple $(T_{\widetilde{v}}(y),\ I_{\widetilde{v}}(y),\ F_{\widetilde{v}}(y))$ that represents the degree of satisfaction from the perspectives of truth, indeterminacy, and falsity.

Chapter 6: Neutrosophic Logic in BioChemistry

In this chapter, we examine the concept of *Neutrosophic Logic in BioChemistry*. The discussion explores how neutrosophic logic—characterized by degrees of truth, indeterminacy, and falsity—can be applied to biochemical systems, where reactions, molecular interactions, and pathway regulations often involve uncertainty and incomplete information.

6.1 Neutrosophic Genetics

Genetics studies heredity and variation, analyzing DNA, genes, chromosomes, and evolutionary mechanisms shaping traits across populations and generations over time [243–245]. Neutrosophic Genetics models inheritance using triplets for truth, indeterminacy, and falsity of gene expression, linkage, and phenotype mapping under uncertainty [246–248].

Definition 6.1 (Neutrosophic Genetics)**.** [246] Let $\mathcal{O}$ be a (nonempty) population of organisms, $\mathcal{G}$ a space of genotypes (or loci), and $\mathcal{E}$ an environment. A *neutrosophic genetic assessment* is any map

$$\Xi:\ \mathcal{O}\times\mathcal{G}\times\mathcal{E}\longrightarrow[0,1]^3,\qquad (o,g,e)\longmapsto\big(T(o,g,e),\ I(o,g,e),\ F(o,g,e)\big),$$

where T is the degree of *evolutionary benefit* (truth), I the degree of *neutrality/indeterminacy*, and F the degree of *involution/harm* (falsity), with the optional normalization $T+I+F\le 1$. Aggregate (individual-, locus-, or population-level) evaluations are obtained by any monotone t–norm/t–conorm based reducer on $[0,1]^3$.

Example 6.2 (Heterozygote Advantage for Malaria: Sickle-Cell Trait (Human Genetics))**.** Let $\mathcal{O}$ be a human population, $\mathcal{G}=\{\mathrm{AA},\mathrm{AS},\mathrm{SS}\}$ the β-globin genotypes, and let the environment $\mathcal{E}$ be indexed by $e=(m,h,\sigma)$ where $m\in[0,1]$ quantifies malaria intensity (e.g., entomological inoculation rate, normalized), $h\in[0,1]$ measures health–care adequacy (1 = excellent), and $\sigma\in[0,1]$ encodes contextual uncertainty (migration, diagnosis, reporting). For the sickle-cell trait $g=\mathrm{AS}$, define

$$\Xi(o,\mathrm{AS},e)=\big(T,I,F\big)\quad\text{with}\quad T=(1-\sigma)\,b(m),\ \ I=\sigma,\ \ F=(1-\sigma)\,s(h),$$

where $b(m):=m$ models *protection against severe malaria* increasing with m, and $s(h):=s_0\,(1-h)$ models *sickle-related harm* decreasing with access to care (baseline $s_0\in(0,1]$). Numerical instance: take $m=0.8$, $h=0.6$, $s_0=0.4$, $\sigma=0.15$. Then

$$T=0.85\times 0.8=0.68,\qquad F=0.85\times(0.4\times 0.4)=0.136,\qquad I=0.15.$$

Interpretation: in a high-malaria, moderate-care setting, AS shows a *high neutrosophic benefit* (anti-malarial effect) with a *nonzero harm* component (sickling risk) and *explicit uncertainty* from environmental variability. Aggregation to the population level can be done by a chosen t–norm/t–conorm over individuals and regions.

Example 6.3 (Antibiotic Resistance Tradeoff: Plasmid-Borne β-Lactamase (Microbial Genetics))**.** Let $\mathcal{O}$ be a bacterial population (e.g., *E. coli*), $\mathcal{G}=\{g_0,g_R\}$ denote absence/presence of a plasmid carrying a β-lactamase gene, and let $\mathcal{E}$ be indexed by $e=(a,c,\sigma)$, where $a\in[0,1]$ is the fraction of time local antibiotic concentration exceeds the MIC (effective exposure), $c\in[0,1]$ is the metabolic burden (fitness cost) of carriage in antibiotic-free conditions, and $\sigma\in[0,1]$ captures uncertainty (copy-number fluctuation, compensatory mutations, biofilm shielding). For the resistant genotype $g=g_R$, set

$$\Xi(o,g_R,e)=\big(T,I,F\big)\quad\text{with}\quad T=(1-\sigma)\,a,\ \ I=\sigma,\ \ F=(1-\sigma)\,(1-a)\,c.$$

Numerical instance: $a=0.7$ (frequent exposure), $c=0.3$ (moderate cost), $\sigma=0.2$ (heterogeneous niches). Then

$$T=0.8\times 0.7=0.56,\qquad F=0.8\times(0.3\times 0.3)=0.072,\qquad I=0.2.$$

Interpretation: under sustained antibiotic pressure, resistance yields *substantial neutrosophic benefit* (T), the *harm* (F) from metabolic burden is curtailed by exposure, while *indeterminacy* (I) accounts for ecological/physiological variability. In antibiotic-free wards (a small), the formula flips the balance by increasing F and reducing T, thus capturing the classic cost-of-resistance tradeoff.

6.2 Neutrosophic Mutation

Mutation introduces genetic changes via replication errors, recombination, transposition, or environmental damage, creating new alleles influencing phenotype and evolution dynamics (cf. [249, 250]). Neutrosophic Mutation assigns to each mutational event triplet degrees for beneficial truth, ambiguous indeterminacy, and deleterious falsity across environments, contexts [246].

Definition 6.4 (Neutrosophic Mutation)**.** [246] Fix $o \in \mathcal{O}$, $g \in \mathcal{G}$, $e \in \mathcal{E}$. A *mutation event* is assigned a triplet

$$\mu(o,g,e) \;:=\; \big(T_\mu(o,g,e),\, I_\mu(o,g,e),\, F_\mu(o,g,e)\big) \in [0,1]^3$$

interpreted as degrees of *positive* (beneficial/adaptive), *neutral* (no effective change or undecidable impact), and *negative* (deleterious/maladaptive) mutation, respectively. Given thresholds $\theta_T^+, \theta_I^0, \theta_F^- \in (0,1]$, we may classify

$$\text{positive if } T_\mu \ge \theta_T^+, \quad \text{neutral if } I_\mu \ge \theta_I^0, \quad \text{negative if } F_\mu \ge \theta_F^-,$$

allowing ties/overlaps when several inequalities hold (reflecting partial truth of multiple statuses). Population summaries are obtained by aggregating $\{\mu(o,g,e)\}$ across (o,g) by weighted means or t–norm/s–conorm mixers.

Example 6.5 (CCR5-Δ32 in Humans: Protection vs. Susceptibility)**.** Let o be a human carrier of the CCR5-Δ32 mutation (heterozygote or homozygote), g ="CCR5-Δ32", and let the environment be $e = (p_{\mathrm{HIV}}, p_{\mathrm{WNV}}, \sigma)$, where $p_{\mathrm{HIV}} \in [0,1]$ quantifies HIV/smallpox–like exposure pressure, $p_{\mathrm{WNV}} \in [0,1]$ quantifies West Nile virus exposure risk, and $\sigma \in [0,1]$ encodes contextual uncertainty (migration, coinfections, clinical access). Define the neutrosophic mutation label

$$\mu(o,g,e) = \big(T_\mu,\, I_\mu,\, F_\mu\big), \qquad T_\mu = (1-\sigma)\, p_{\mathrm{HIV}}, \quad F_\mu = (1-\sigma)\, p_{\mathrm{WNV}}, \quad I_\mu = \sigma.$$

Numerical instance: $p_{\mathrm{HIV}} = 0.7$, $p_{\mathrm{WNV}} = 0.2$, $\sigma = 0.15$ gives

$$T_\mu = 0.85 \times 0.7 = 0.595, \quad F_\mu = 0.85 \times 0.2 = 0.17, \quad I_\mu = 0.15.$$

Interpretation: in settings with substantial HIV (or historical smallpox) pressure and moderate uncertainty, CCR5-Δ32 carries a *beneficial* degree (reduced infection risk), a smaller *deleterious* degree (heightened WNV susceptibility), and explicit *indeterminacy*.

Example 6.6 (SARS-CoV-2 Spike E484K: Immune Evasion vs. Fitness Cost)**.** Let o be a viral lineage bearing the spike mutation E484K, g ="E484K", and let the environment be $e = (a, c, \sigma)$, where $a \in [0,1]$ measures antibody/immune pressure (from prior infection/vaccination or plasma therapy), $c \in [0,1]$ captures replication/affinity cost in the host (higher means costlier), and $\sigma \in [0,1]$ encodes epidemiological/measurement uncertainty (sampling noise, cofactors). Assign

$$\mu(o,g,e) = \big(T_\mu,\, I_\mu,\, F_\mu\big), \qquad T_\mu = (1-\sigma)\, a, \quad F_\mu = (1-\sigma)\, c, \quad I_\mu = \sigma.$$

Numerical instance: $a = 0.8$ (strong immune pressure), $c = 0.25$ (moderate cost), $\sigma = 0.2$ yields

$$T_\mu = 0.8 \times 0.8 = 0.64, \quad F_\mu = 0.8 \times 0.25 = 0.20, \quad I_\mu = 0.20.$$

Interpretation: under high antibody pressure, E484K shows a *beneficial* neutrosophic degree (escape), a *deleterious* degree from fitness costs, and *indeterminacy* reflecting uncertain transmission context and measurement variability.

6.3 Neutrosophic Speciation

Neutrosophic Speciation evaluates candidate species boundaries by triplets on reproductive isolation truth, taxonomic indeterminacy, and ecological divergence falsity over time [246].

Definition 6.7 (Neutrosophic Speciation)**.** [246] Let Sp denote a species (as a dynamic lineage). Define its *speciation status* at environment $e \in \mathcal{E}$ by

$$\Sigma(\mathsf{Sp}, e) \;:=\; \big(T_{\mathrm{spec}}(\mathsf{Sp}, e),\, I_{\mathrm{cont}}(\mathsf{Sp}, e),\, F_{\mathrm{ext}}(\mathsf{Sp}, e)\big) \in [0,1]^3,$$

where T_{spec} is the degree of *speciation tendency* (formation of a new species), I_{cont} the degree of *continuation* (stasis as the same species) interpreted neutrosophically as neutral/indeterminate between speciation and extinction, and F_{ext} the degree of *extinction* tendency. Given decision thresholds $(\tau_T^\uparrow, \tau_I^\uparrow, \tau_F^\uparrow) \in (0,1]^3$, one may declare

$$\text{“speciation likely” if } T_{\mathrm{spec}} \ge \tau_T^\uparrow, \quad \text{“continuation” if } I_{\mathrm{cont}} \ge \tau_I^\uparrow, \quad \text{“extinction risk” if } F_{\mathrm{ext}} \ge \tau_F^\uparrow,$$

with mixed outcomes allowed (overlapping degrees). Time dependence $t \mapsto \Sigma(\mathsf{Sp}, e_t)$ is handled componentwise.

Example 6.8 (Crater–Lake Cichlids: Assortative Mating vs. Gene Flow)**.** Consider two incipient cichlid morphs in a crater lake. Let $a \in [0,1]$ be assortative–mating strength (sensory drive), $d \in [0,1]$ ecological divergence (diet/habitat), $m \in [0,1]$ effective gene flow (connectivity), $p \in [0,1]$ predation pressure, $s_h \in [0,1]$ habitat stability, and $\sigma \in [0,1]$ a measurement/context uncertainty mass. Define the neutrosophic speciation label

$$\Sigma = \big(T_{\mathrm{spec}},\, I_{\mathrm{cont}},\, F_{\mathrm{ext}}\big), \quad T_{\mathrm{spec}} = (1-\sigma)\, \min\{1,\, a\, d\, (1-m)\},$$

$$I_{\mathrm{cont}} = \min\{1,\, \sigma + (1-\sigma)\, m\, (1-a)\}, \qquad F_{\mathrm{ext}} = (1-\sigma)\, p\, (1-s_h).$$

Numerical instance: $a = 0.7$, $d = 0.8$, $m = 0.2$, $p = 0.1$, $s_h = 0.9$, $\sigma = 0.15$ give

$$T_{\mathrm{spec}} = 0.85 \times \min\{1, 0.7 \cdot 0.8 \cdot 0.8\} = 0.85 \times 0.448 = 0.381,$$

$$I_{\mathrm{cont}} = 0.15 + 0.85 \times 0.2 \times 0.3 = 0.201, \qquad F_{\mathrm{ext}} = 0.85 \times 0.1 \times 0.1 = 0.0085\ (\approx 0.009).$$

Interpretation: strong assortative mating and niche divergence promote speciation (T_{spec}), residual gene flow sustains lineage continuation/ambiguity (I_{cont}), while low predation and stable habitat keep extinction tendency small (F_{ext}).

Example 6.9 (Ring–Species Salamanders (Ensatina): Isolation vs. Introgression)**.** Around California's Central Valley, *Ensatina* forms a geographic ring with terminal populations showing partial reproductive isolation. Let $g \in [0,1]$ quantify geographic isolation (barrier strength), $h \in [0,1]$ hybrid incompatibility (postzygotic isolation), $i \in [0,1]$ introgression rate along the ring, $w \in [0,1]$ environmental risk (e.g., wildfire/climate), and $\sigma \in [0,1]$ uncertainty. Set

$$T_{\text{spec}} = (1-\sigma)\,\min\{1,\ g\,h\}, \qquad I_{\text{cont}} = \min\{1,\ \sigma + (1-\sigma)\,i\,(1-h)\}, \qquad F_{\text{ext}} = (1-\sigma)\,w.$$

Numerical instance: $g = 0.8$, $h = 0.6$, $i = 0.4$, $w = 0.2$, $\sigma = 0.2$ yield

$$T_{\text{spec}} = 0.8 \times \min\{1, 0.48\} = 0.384, \quad I_{\text{cont}} = 0.2 + 0.8 \times 0.4 \times 0.4 = 0.328, \quad F_{\text{ext}} = 0.8 \times 0.2 = 0.16.$$

Interpretation: substantial isolation and incompatibility support incipient speciation (T_{spec}), ongoing introgression sustains continuation/ambiguity (I_{cont}), and regional hazards contribute a nonzero extinction tendency (F_{ext}).

6.4 Neutrosophic Coevolution

Neutrosophic Coevolution quantifies interacting lineages with triplets measuring adaptive reciprocity truth, uncertain neutrality indeterminacy, and antagonistic conflict falsity across generations.

Definition 6.10 (Neutrosophic Coevolution)**.** [246] For two species $\mathsf{Sp}_1, \mathsf{Sp}_2$ interacting within $e \in \mathcal{E}$, define the *coevolution triplet*

$$C(\mathsf{Sp}_1, \mathsf{Sp}_2\,;\, e) \ := \ \big(T_{\text{coop}},\ I_{\text{neut}},\ F_{\text{conf}}\big) \in [0,1]^3,$$

where T_{coop} measures *cooperative/beneficial* (mutualism/commensalism) influence, F_{conf} measures *conflict/antagonism* (competition/predation/parasitism), and I_{neut} measures *neutrality/uncertainty* (no effect or undecidable effect). A (directed) kernel Λ may be specified at trait level; for trait-sets A_i of Sp_i set

$$T_{\text{coop}} = \sup_{a_1 \in A_1,\, a_2 \in A_2} t\big(\lambda_T(a_1,a_2),\ \lambda_T(a_2,a_1)\big), \quad F_{\text{conf}} = \sup_{a_1,a_2} t\big(\lambda_F(a_1,a_2),\ \lambda_F(a_2,a_1)\big),$$

and

$$I_{\text{neut}} = \sup_{a_1,a_2} s\big(\lambda_I(a_1,a_2),\ \lambda_I(a_2,a_1)\big),$$

with t a t–norm and s its dual conorm. Environmental dynamics e_t induce time-varying C_t, capturing shifting degrees of cooperation, neutrality, and conflict.

Example 6.11 (Plant–Pollinator Coevolution under Mismatch and Nectar Robbing)**.** Let Sp_1 be a hummingbird and Sp_2 a tubular flower species in environment e. Take $t = \min$ and $s = \max$ as t–norm/t–conorm. Traits:

$$A_1 = \{\text{beak: } S, L\}, \qquad A_2 = \{\text{corolla: } s, d\}.$$

Define the (symmetric) neutrosophic interaction kernel $\Lambda = (\lambda_T, \lambda_I, \lambda_F)$ by

$\lambda_T(a_1,a_2)$	s	d
S	0.80	0.20
L	0.25	0.90

$\lambda_I(a_1,a_2)$	s	d
S	0.15	0.20
L	0.20	0.10

$\lambda_F(a_1,a_2)$	s	d
S	0.05	0.50
L	0.30	0.05

(higher λ_F on mismatches models nectar robbing or wasted visits; λ_I captures weather/phenology ambiguity). Then

$$T_{\text{coop}} = \sup_{a_1 \in A_1, a_2 \in A_2} \min\{\lambda_T(a_1,a_2), \lambda_T(a_2,a_1)\} = \max\{0.80, 0.20, 0.25, 0.90\} = 0.90,$$

$$F_{\text{conf}} = \sup_{a_1,a_2} \min\{\lambda_F(a_1,a_2), \lambda_F(a_2,a_1)\} = \max\{0.05, 0.50, 0.30, 0.05\} = 0.50,$$

$$I_{\text{neut}} = \sup_{a_1,a_2} \max\{\lambda_I(a_1,a_2), \lambda_I(a_2,a_1)\} = \max\{0.15, 0.20, 0.20, 0.10\} = 0.20.$$

Interpretation: tight morphology match (L, d) yields high cooperative benefit (T_{coop}), mismatches admit conflict via robbing (F_{conf}), and seasonal/weather uncertainty maintains moderate indeterminacy (I_{neut}).

Example 6.12 (Cleaner–Client Mutualism with Cheating and Sanctions)**.** Let Sp_1 be a cleaner wrasse and Sp_2 a reef client fish. Choose $t = \min$, $s = \max$. Traits:

$$A_1 = \{\text{Cleaner: Honest } (H),\ \text{Cheat } (C)\}, \quad A_2 = \{\text{Client: Tolerate } (T),\ \text{Sanction } (S)\}.$$

Specify (possibly asymmetric) kernel values:

Truth / cooperative benefit (parasite removal, access to clients):

$\lambda_T(a_1 \to a_2)$	T	S
H	0.90	0.85
C	0.10	0.05

$\lambda_T(a_2 \to a_1)$	H	C
T	0.80	0.30
S	0.75	0.20

Falsity / conflict (mucus nibbles, chases):

$\lambda_F(a_1 \to a_2)$	T	S
H	0.05	0.05
C	0.70	0.60

$\lambda_F(a_2 \to a_1)$	H	C
T	0.05	0.20
S	0.10	0.60

Indeterminacy (observation noise, seasonality, partner switching):

$\lambda_I(a_1 \leftrightarrow a_2)$	T	S
H	0.05	0.08
C	0.12	0.20

Compute

$$T_{\text{coop}} = \sup_{a_1,a_2} \min\{\lambda_T(a_1 \to a_2), \lambda_T(a_2 \to a_1)\} = \max\{\min(0.90, 0.80), \min(0.85, 0.75), \ldots\} = 0.80,$$

$$F_{\text{conf}} = \sup_{a_1,a_2} \min\{\lambda_F(a_1 \to a_2), \lambda_F(a_2 \to a_1)\} = \max\{\min(0.70, 0.20), \min(0.60, 0.60), \ldots\} = 0.60,$$

$$I_{\text{neut}} = \sup_{a_1,a_2} \max\{\lambda_I(a_1 \leftrightarrow a_2)\} = \max\{0.05, 0.08, 0.12, 0.20\} = 0.20.$$

Interpretation: honest cleaning with tolerant clients drives strong mutual benefit (T_{coop}); cheating met by sanctions induces substantial conflict (F_{conf}); fluctuating ecology and monitoring limit knowledge, sustaining indeterminacy (I_{neut}).

Chapter 7: Neutrosophic Physics

In this chapter, we examine the concept of Neutrosophic Physics, exploring how neutrosophic logic—based on truth, indeterminacy, and falsity degrees—can be applied to model uncertainty, duality, and indeterminate states in physical systems.

7.1 Neutrosophic Physics

Neutrosophic physics models physical entities with simultaneous truth, indeterminacy, and falsity degrees, accommodating contradictory states, hybrid phases, and context-dependent behaviors.

Definition 7.1 (Neutrosophic Physics)**.** Let $\langle A, \text{anti } A, \text{neut } A\rangle$ denote, respectively, a physical entity (concept, law, state, property), its opposite, and a spectrum of neutral/intermediate forms. *Neutrosophic physics* studies physical objects, processes, and theories that can simultaneously instantiate degrees of $\langle A\rangle$ and $\langle \text{anti } A\rangle$ (and possibly $\langle \text{neut } A\rangle$), modeled by neutrosophic triplets attached to propositions or states. Typical examples include mixed or hybrid phases, contradictory/ambiguous states, and context–dependent behaviors (e.g. matter/antimatter mixtures or other multi-status entities).

Example 7.2 (Neutrosophic Physics: Mixed–Phase Ice Slurry)**.** Consider the proposition A: "the specimen is *solid*." A calorimetric assay of a near–0°C ice slurry yields estimated phase fractions: crystalline (solid) 0.62, liquid 0.32, and unresolved slush/measurement uncertainty 0.06. Attach to A the neutrosophic truth value

$$v_A = (T, I, F) = (0.62,\ 0.06,\ 0.32).$$

Here T reflects the solid fraction supporting A, F reflects the liquid fraction contradicting A, and I captures indeterminacy from mushy microstructure and sensor noise. Since $T > 0$ and $F > 0$, the specimen *simultaneously* instantiates degrees of A and $\neg A$, making it a typical neutrosophic physical state (mixed status under controlled ambiguity).

7.2 Neutrosophic Newtonian Compliance

Newtonian compliance quantifies linear viscoelastic response: strain per unit stress increases proportionally with time, slope inversely equals viscosity for fluids. Neutrosophic Newtonian compliance represents strain–stress compliance by triplets: truth of Newtonian fit, indeterminacy from noise/heterogeneity, falsity indicating systematic non-Newtonian deviations.

Definition 7.3 (Neutrosophic Newtonian Compliance (Mechanics))**.** Consider a point mass $m > 0$ with measured net force $F \in \mathbb{R}^d$ and acceleration $a \in \mathbb{R}^d$. Let the residual $r := F - ma$ and the alignment

$$\alpha := \begin{cases} \dfrac{1}{2}\left(1 + \dfrac{F \cdot a}{\|F\|\,\|a\|}\right), & \text{if } F \neq 0 \text{ and } a \neq 0, \\ 1, & \text{otherwise,} \end{cases} \qquad \alpha \in [0, 1].$$

For a tolerance scale $\tau > 0$, define consistency $\kappa := \dfrac{1}{1 + \|r\|/\tau} \in (0, 1]$. The *neutrosophic compliance* of $F = ma$ is

$$T_{\text{dyn}} := \kappa\,\alpha, \qquad F_{\text{dyn}} := (1 - \kappa)\,(1 - \alpha), \qquad I_{\text{dyn}} := \text{clip}\big(1 - T_{\text{dyn}} - F_{\text{dyn}}\big).$$

Here T_{dyn} is high when the residual is small and F and a are well-aligned; F_{dyn} grows when the residual is large and F opposes a; I_{dyn} collects the remaining ambiguity (noise, unmodeled forces).

Example 7.4 (Neutrosophic Newtonian Compliance: Quadcopter Under Gusts)**.** A drone of mass $m = 2.0\,\text{kg}$ experiences measured net force $F = (5,\ 1,\ 0)\,\text{N}$ and acceleration $a = (2.4,\ 0.3,\ 0)\,\text{m/s}^2$ (in a world frame). The residual is

$$r := F - ma = (0.2,\ 0.4,\ 0), \quad \|r\| = \sqrt{0.2} \approx 0.447.$$

With tolerance scale $\tau = 1$, the consistency factor is $\kappa = \dfrac{1}{1 + \|r\|/\tau} \approx 0.691$. The force–acceleration alignment is

$$\alpha = \tfrac{1}{2}\left(1 + \frac{F \cdot a}{\|F\|\,\|a\|}\right) = \tfrac{1}{2}\left(1 + \frac{12.3}{\sqrt{26}\,\sqrt{5.85}}\right) \approx 0.999.$$

The neutrosophic compliance triplet (for $F = ma$) is

$$T_{\text{dyn}} = \kappa\,\alpha \approx 0.690, \qquad F_{\text{dyn}} = (1 - \kappa)(1 - \alpha) \approx 4.12 \times 10^{-4}, \qquad I_{\text{dyn}} = 1 - T_{\text{dyn}} - F_{\text{dyn}} \approx 0.309.$$

Thus the data *largely* support Newton's law (high T_{dyn}), with a tiny falsity due to anti-alignment and residual, and a moderate indeterminacy reflecting unmodeled disturbances (gusts) and sensor limits.

7.3 Neutrosophic Magnetic Field

A magnetic field is a vector field produced by moving charges and magnets, exerting forces on charges and magnetic dipoles [251–253]. A neutrosophic magnetic field assigns each space-time point a magnetic vector plus truth, indeterminacy, and falsity degrees quantifying measurement uncertainty [254].

Definition 7.5 (Neutrosophic Magnetic Field)**.** [254] Let $D \subseteq \mathbb{R}^3$ be a (nonempty) spatial domain and let $I \subseteq \mathbb{R}$ be a time interval. A *neutrosophic magnetic field* on $D \times I$ is a mapping

$$\mathcal{B} \;:\; D \times I \longrightarrow \big([0,1]^3 \times \mathbb{R}^3\big), \qquad (x,t) \longmapsto \big((T_B(x,t),\, I_B(x,t),\, F_B(x,t)),\, \vec{B}(x,t)\big),$$

with the following components and conditions.

1. $\vec{B}(x,t) \in \mathbb{R}^3$ is the (classical) magnetic induction/intensity vector that would be observed at point $x \in D$ and time $t \in I$ under an ideal, fully determined physical state.
2. $T_B, I_B, F_B : D \times I \to [0,1]$ are, respectively, the *truth*, *indeterminacy*, and *falsity* degrees attached to the statement

   "the magnetic field at (x,t) is $\vec{B}(x,t)$ (within the modeling tolerance)".

   For every $(x,t) \in D \times I$ these degrees are independent and satisfy

   $$0 \;\le\; T_B(x,t) + I_B(x,t) + F_B(x,t) \;\le\; 3.$$

3. $T_B(x,t)$ quantifies how strongly the available information (sensor readings, theoretical model, boundary conditions, source currents) supports $\vec{B}(x,t)$ as the effective magnetic field.
4. $I_B(x,t)$ quantifies the neutral/unknown/indeterminate part, arising for example from noisy or incomplete measurements, unmodeled micro–currents, partially specified material properties, or conflicting field extrapolations.
5. $F_B(x,t)$ quantifies how strongly the available information refutes $\vec{B}(x,t)$ as the effective magnetic field (e.g. alternative numerical solution, incompatible polarization, or contradictory boundary data).

When $I_B(x,t) = 0$ and $F_B(x,t) = 1 - T_B(x,t)$ for all (x,t), $\mathcal{B}$ reduces to a fuzzy magnetic field; when, in addition, $T_B(x,t) + F_B(x,t) = 1$ for all (x,t), it reduces further to the classical (crisp) magnetic field.

Remark 7.6 (Link with neutrosophic magnetic polarity)**.** In the neutrosophic physics setting (cf. Section 7.4 *Neutrosophic Magnetic Polarity*), a magnetic source may have a triplet-valued polarity (T_P, I_P, F_P). If a field point (x,t) is influenced by such a source, one may impose the consistency condition

$$I_B(x,t) \;\ge\; I_P,$$

i.e. indeterminacy in the magnetic polarity propagates to (and lower–bounds) the indeterminacy of the field at the observation point.

Example 7.7 (Solenoid with uncertain current)**.** Consider a long solenoid whose axis coincides with the z–axis. Ideally, at time t the interior magnetic field is $\vec{B}_{\text{ideal}}(x,t) = B_0(t)\,\hat{z}$, where $B_0(t) = \mu_0 n I(t)$, n is the turn density, and $I(t)$ is the current. Suppose now that the current measurement is partially unreliable:

$$T_I(t) = 0.7, \qquad I_I(t) = 0.2, \qquad F_I(t) = 0.3.$$

On the cylinder $D = \{(r,\theta,z) : r < R\}$ one may define the neutrosophic magnetic field

$$\mathcal{B}(x,t) = \big((T_B(x,t), I_B(x,t), F_B(x,t)),\, \vec{B}_{\text{ideal}}(x,t)\big),$$

with

$$T_B(x,t) = 0.7, \qquad I_B(x,t) = 0.2, \qquad F_B(x,t) = 0.3, \qquad \vec{B}_{\text{ideal}}(x,t) = B_0(t)\,\hat{z}.$$

Thus the direction and formula of the field are known (so we keep $\vec{B}_{\text{ideal}}$), but its *validity* at (x,t) is neutrosophically qualified by $(0.7, 0.2, 0.3)$, which records support, incompleteness, and contradiction, respectively.

7.4 Neutrosophic Wave Coherence and Interference

Wave coherence and interference describe phase relations producing constructive or destructive patterns, quantifying field correlation across space, time, and frequency(cf. [255–259]). Neutrosophic wave coherence models phase correlation with triplets: truth of coherence, indeterminacy from noise, falsity representing decoherence or contradictory signals.

Definition 7.8 (Neutrosophic Wave Coherence and Interference (Waves))**.** Let u_1, u_2 be real (or complex) waveforms on a window $[t_0, t_0 + T]$. Write

$$\langle u_1, u_2 \rangle := \int_{t_0}^{t_0+T} u_1(t)\,\overline{u_2(t)}\,dt, \qquad \|u\| := \sqrt{\langle u, u \rangle}.$$

Define the (windowed) *coherence* $C := \dfrac{|\langle u_1, u_2 \rangle|}{\|u_1\|\,\|u_2\|} \in [0, 1]$ and phase difference $\Delta\varphi := \arg\langle u_1, u_2 \rangle$. The *neutrosophic interference profile* is

$$T_{\text{wave}} := C\,\frac{1 + \cos\Delta\varphi}{2}, \qquad F_{\text{wave}} := C\,\frac{1 - \cos\Delta\varphi}{2}, \qquad I_{\text{wave}} := 1 - C.$$

Thus T_{wave} captures constructive interference, F_{wave} destructive interference, and I_{wave} the unexplained (incoherent) portion.

Example 7.9 (Field–programmable microwave interferometer: wave coherence on a link)**.** Two RF channels u_1, u_2 carry narrowband signals between synchronized radios over a $T = 10$ ms window. The windowed coherence is measured $C = 0.80$ and the estimated phase offset is $\Delta\varphi = 30^\circ$ (constructive). Using

$$T_{\text{wave}} = C\,\tfrac{1+\cos\Delta\varphi}{2}, \quad F_{\text{wave}} = C\,\tfrac{1-\cos\Delta\varphi}{2}, \quad I_{\text{wave}} = 1 - C,$$

with $\cos 30^\circ = 0.866$, we obtain

$$T_{\text{wave}} \approx 0.800 \times 0.933 = 0.746, \quad F_{\text{wave}} \approx 0.800 \times 0.067 = 0.054, \quad I_{\text{wave}} = 0.200.$$

Interpretation: strong constructive interference (≈ 0.75), little destructive content (≈ 0.05), and 20% incoherence due to multipath/noise.

7.5 Neutrosophic Kirchhoff Consistency

Neutrosophic Kirchhoff consistency assigns truth, indeterminacy, and falsity degrees to Kirchhoff equations, reflecting component tolerances, noise, modeling ambiguity, and violations (cf. [260–263])

Definition 7.10 (Neutrosophic Kirchhoff Consistency (Electric Circuits))**.** Consider a lumped network with node set $\mathcal{N}$. For node $j \in \mathcal{N}$ let $S_j := \sum_{e \in \text{inc}(j)} I_e$ be the signed sum of branch currents entering j (measured or estimated). Fix a tolerance $\tau_j > 0$ and define the *KCL truth degree*

$$T_{\text{kcl}}(j) := \exp\!\big(-|S_j|/\tau_j\big) \in (0, 1].$$

Let $\chi_j \in [0, 1]$ be a *conflict index* at j (e.g. the fraction of incident branches whose measured current directions disagree with those predicted from the present voltage estimates). Then set

$$F_{\text{kcl}}(j) := \chi_j\big(1 - T_{\text{kcl}}(j)\big), \qquad I_{\text{kcl}}(j) := \text{clip}\big(1 - T_{\text{kcl}}(j) - F_{\text{kcl}}(j)\big).$$

Thus T_{kcl} quantifies how well KCL is satisfied, F_{kcl} captures systematic violations (conflicting flows), and I_{kcl} represents sensor noise, missing data, or unmodeled parasitics. Network-level neutrosophic consistency is obtained by aggregating $(T_{\text{kcl}}, I_{\text{kcl}}, F_{\text{kcl}})$ over $j \in \mathcal{N}$ via a chosen t–norm/t–conorm.

Example 7.11 (Circuit bring-up: neutrosophic KCL check at a sensor node)**.** At node j of a mixed-signal PCB, the signed sum of measured branch currents is $S_j = +0.15$ A. With tolerance $\tau_j = 0.10$ A,

$$T_{\text{kcl}}(j) = \exp(-|S_j|/\tau_j) = \exp(-1.5) \approx 0.223.$$

A conflict index $\chi_j = 0.60$ (three of five branches disagree in direction) yields

$$F_{\text{kcl}}(j) = \chi_j(1 - T_{\text{kcl}}) \approx 0.60 \times 0.777 = 0.466, \quad I_{\text{kcl}}(j) = 1 - T_{\text{kcl}} - F_{\text{kcl}} \approx 0.311.$$

Conclusion: KCL violations are primarily systematic (high F), with moderate residual uncertainty (I).

7.6 Neutrosophic Optical Transfer Compliance

Optics studies light's generation, propagation, and interaction with matter, including reflection, refraction, diffraction, polarization, imaging, optical materials, scattering, and absorption [264–266]. Neutrosophic optical transfer compliance evaluates MTF/OTF agreement using truth, indeterminacy, falsity degrees across spatial frequencies, accounting for noise and aberrations.

Definition 7.12 (Neutrosophic Optical Transfer Compliance (Optics))**.** Let an imaging system have measured modulation transfer $M(f)$ and specification $M_{\text{spec}}(f)$. Fix a spatial frequency $f_0 > 0$ of interest and a (relative) measurement uncertainty $u_M \in [0, 1]$ estimated from the SNR or repeatability of $M(f_0)$. Set

$$c := \text{clip}\left(\frac{M(f_0)}{M_{\text{spec}}(f_0)}\right), \qquad u := u_M.$$

The *neutrosophic compliance* of the optical transfer at f_0 is

$$T_{\text{opt}} := c(1-u), \qquad I_{\text{opt}} := u, \qquad F_{\text{opt}} := (1-c)(1-u).$$

Hence $T_{\text{opt}} + I_{\text{opt}} + F_{\text{opt}} = 1$: truth measures spec satisfaction, falsity measures shortfall, and indeterminacy captures measurement/coherence uncertainties (e.g., speckle, phase retrieval residuals).

Example 7.13 (Imaging lens QA: modulation transfer at 50 lp/mm)**.** Measured MTF at $f_0 = 50$ lp/mm is $M(f_0) = 0.32$; the specification requires $M_{\text{spec}}(f_0) = 0.40$. Let relative measurement uncertainty be $u = 0.15$. With $c = \text{clip}\big(M/M_{\text{spec}}\big) = 0.80$,

$$T_{\text{opt}} = c(1-u) = 0.80 \times 0.85 = 0.680, \quad I_{\text{opt}} = u = 0.150, \quad F_{\text{opt}} = (1-c)(1-u) = 0.20 \times 0.85 = 0.170.$$

Hence the lens is close to spec (truth ≈ 0.68), with modest shortfall and quantified metrology uncertainty.

7.7 Neutrosophic Second-Law Consistency (Thermodynamics)

Thermodynamics studies energy, heat, work, and entropy in macroscopic systems, establishing equilibrium, state variables, transformations, and fundamental laws governing processes [267–270]. Neutrosophic second-law consistency grades entropy-production statements by truth, indeterminacy, falsity, capturing measurement uncertainty, model ambiguity, and violations across spatiotemporal scales.

Definition 7.14 (Neutrosophic Second-Law Consistency (Thermodynamics))**.** Let $\dot{S}_{\text{prod}}$ denote the (instantaneous or averaged) entropy production rate and $\widehat{\dot{S}}_{\text{prod}}$ its estimator with standard uncertainty $\sigma > 0$. Under a Gaussian approximation, the probability that the second law holds is

$$p \;:=\; \mathbb{P}(\dot{S}_{\text{prod}} \geq 0) \;=\; \Phi\left(\frac{\widehat{\dot{S}}_{\text{prod}}}{\sigma}\right),$$

where Φ is the standard normal CDF. Define a *symmetry uncertainty* $u := 2\min\{p, 1-p\} \in [0, 1]$. The neutrosophic assessment is

$$T_{\text{2nd}} := p(1-u), \qquad F_{\text{2nd}} := (1-p)(1-u), \qquad I_{\text{2nd}} := u,$$

so that $T_{\text{2nd}} + I_{\text{2nd}} + F_{\text{2nd}} = 1$. Large positive estimates yield $T_{\text{2nd}} \approx 1$; negative estimates inflate F_{2nd}; near-zero estimates (with high ambiguity) raise I_{2nd}.

Example 7.15 (Entropy production in a microfluidic heat exchanger)**.** A trajectory estimator yields $\widehat{\dot{S}}_{\text{prod}} = 0.05$ (nondimensional units) with standard uncertainty $\sigma = 0.06$. Then

$$p = \Phi\left(\frac{0.05}{0.06}\right) = \Phi(0.833) \approx 0.798, \qquad u = 2\min\{p, 1-p\} \approx 0.405.$$

Therefore

$$T_{\text{2nd}} = p(1-u) \approx 0.798 \times 0.595 = 0.475, \quad F_{\text{2nd}} = (1-p)(1-u) \approx 0.202 \times 0.595 = 0.120, \quad I_{\text{2nd}} \approx 0.405.$$

Interpretation: evidence favors nonnegative entropy production, but proximity to equilibrium inflates indeterminacy.

7.8 Neutrosophic Laminarity (Fluid Mechanics)

Laminarity describes smooth, orderly fluid motion in parallel layers with minimal mixing, characterized by low Reynolds numbers and stable streamlines (cf. [271–275]). Neutrosophic laminarity assigns truth, indeterminacy, falsity degrees to laminar flow, incorporating measurement noise, transitional regimes, and model ambiguity across scales.

Definition 7.16 (Neutrosophic Laminarity (Fluid Mechanics))**.** For an internal flow with bulk speed U, characteristic length L, kinematic viscosity ν, Reynolds number $\mathrm{Re} = UL/\nu$, and turbulence intensity $\mathrm{TI} \in [0,1]$, fix transition thresholds $0 < \mathrm{Re}_1 < \mathrm{Re}_2$ and a maximum tolerable intensity $\mathrm{TI}_{\max} \in (0,1]$. Define laminar compliance

$$c_{\mathrm{Re}} := \mathrm{clip}\left(1 - \frac{\mathrm{Re} - \mathrm{Re}_1}{\mathrm{Re}_2 - \mathrm{Re}_1}\right), \qquad c_{\mathrm{TI}} := \mathrm{clip}\left(1 - \frac{\mathrm{TI}}{\mathrm{TI}_{\max}}\right), \qquad c := c_{\mathrm{Re}}\, c_{\mathrm{TI}}.$$

Let $u \in [0,1]$ encode estimation variability (e.g., coefficient of variation from repeated Re/TI measurements). Set

$$T_{\mathrm{lam}} := c(1-u), \qquad I_{\mathrm{lam}} := u, \qquad F_{\mathrm{lam}} := (1-c)(1-u).$$

Thus T_{lam} grades laminarity, F_{lam} grades turbulence, and I_{lam} captures transitional/measurement ambiguity.

Example 7.17 (HVAC duct flow: laminarity audit)**.** An air stream in a circular duct has $U = 3.0\,\mathrm{m/s}$, $L = 0.06\,\mathrm{m}$, $\nu = 1.0 \times 10^{-5}\,\mathrm{m^2/s}$, so $\mathrm{Re} = UL/\nu = 18{,}000$. Suppose an internal baffle reduces the effective L for onset to $\mathrm{Re}_2 = 2{,}300$ with a laminar guard $\mathrm{Re}_1 = 1{,}500$ (assessment window [1,500, 2,300]). A local flow conditioner yields turbulence intensity $\mathrm{TI} = 0.04$ with $\mathrm{TI}_{\max} = 0.10$. Then

$$c_{\mathrm{Re}} = \mathrm{clip}\left(1 - \frac{\mathrm{Re} - \mathrm{Re}_1}{\mathrm{Re}_2 - \mathrm{Re}_1}\right) = \mathrm{clip}\left(1 - \frac{300}{800}\right) = 0.625, \quad c_{\mathrm{TI}} = \mathrm{clip}\left(1 - \frac{0.04}{0.10}\right) = 0.60,$$

$$c = c_{\mathrm{Re}} c_{\mathrm{TI}} = 0.375, \quad u = 0.10, \quad T_{\mathrm{lam}} = 0.338,\ I_{\mathrm{lam}} = 0.100,\ F_{\mathrm{lam}} = 0.563.$$

Thus the flow is more turbulent than laminar (F large), with limited estimation uncertainty.

7.9 Neutrosophic Gibbs–Equilibrium Consistency (Statistical Mechanics)

Statistical mechanics links microscopic particle ensembles to macroscopic thermodynamics, using probability distributions to predict equilibrium, fluctuations, transport, responses, rates [276–283]. Neutrosophic Gibbs–Equilibrium Consistency assesses equilibrium closeness and stationarity via divergence and stationarity metrics, yielding truth, indeterminacy, and falsity triplets scores.

Definition 7.18 (Neutrosophic Gibbs–Equilibrium Consistency (Statistical Mechanics))**.** Let $\hat{p}$ be the empirical distribution of microstate energies $\{E_i\}$ from a stationary time window and $p_\beta(i) = Z(\beta)^{-1} e^{-\beta E_i}$ the Boltzmann model at inverse temperature β. Compute the Kullback–Leibler divergence $D := D_{\mathrm{KL}}(\hat{p} \| p_\beta) \geq 0$ and a stationarity score $s := \exp(-\mathrm{KS}/\kappa) \in (0,1]$ from the Kolmogorov–Smirnov statistic KS between the first/second halves of the window (scale $\kappa > 0$). Let the sampling uncertainty be $u := 1 - \min\{1, N_{\mathrm{eff}}/N\}$, where N is the total sample size and N_{eff} an effective sample size estimate. With a divergence scale $\tau > 0$, define

$$c := \exp(-D/\tau)\, s, \qquad T_{\mathrm{eq}} := c(1-u), \qquad I_{\mathrm{eq}} := u, \qquad F_{\mathrm{eq}} := (1-c)(1-u).$$

Then T_{eq} measures closeness to a Gibbs equilibrium *and* temporal stationarity, F_{eq} measures structured departures, while I_{eq} captures finite-sample and coverage uncertainty.

Example 7.19 (Molecular Simulation: Assessing Gibbs–Equilibrium Consistency)**.** A molecular–dynamics run of $N = 10{,}000$ Lennard–Jones particles yields an empirical energy distribution $\hat{p}$ over microstates $\{E_i\}$. Fitting the Boltzmann model $p_\beta(i) = Z(\beta)^{-1} e^{-\beta E_i}$ at the observed mean energy gives $D = D_{\mathrm{KL}}(\hat{p} \| p_\beta) = \tau \ln 2$ with a chosen divergence scale $\tau > 0$. Splitting the trajectory in half and computing a Kolmogorov–Smirnov statistic $\mathrm{KS} = \kappa \ln 2$ (with scale $\kappa > 0$) produces a stationarity score $s = \exp(-\mathrm{KS}/\kappa) = \frac{1}{2}$. An effective sample–size analysis returns $N_{\mathrm{eff}} = 8{,}000$, hence $u = 1 - \min\{1, N_{\mathrm{eff}}/N\} = 0.2$. Therefore

$$c = \exp(-D/\tau)\, s = \tfrac{1}{2} \cdot \tfrac{1}{2} = \tfrac{1}{4}, \qquad T_{\mathrm{eq}} = c(1-u) = 0.25 \times 0.8 = 0.20, \quad I_{\mathrm{eq}} = 0.20, \quad F_{\mathrm{eq}} = (1-c)(1-u) = 0.75 \times 0.8 = 0.60.$$

Interpretation: the run shows modest proximity to a Gibbs equilibrium ($T_{\mathrm{eq}} = 0.20$), clear structured departures ($F_{\mathrm{eq}} = 0.60$), and sampling/coverage uncertainty of 0.20.

7.10 Neutrosophic Probabilistic Hazard Consistency (Earth Science)

Earth science investigates Earth's systems—geosphere, hydrosphere, atmosphere, biosphere—analyzing processes, interactions, and changes using observations, experiments, and modeling across spatial scalesand falsity components. [284–287] A neutrosophic probabilistic hazard consistency metric evaluates forecast calibration, threshold ambiguity, and data uncertainty via truth, indeterminacy,

Definition 7.20 (Neutrosophic Probabilistic Hazard Consistency (Earth Science))**.** Let a site s have a forecast that the ground–motion threshold A_{thr} will be exceeded in a verification window W with probability $q \in [0,1]$. At the end of W, let $y \in \{0,1\}$ be the observed outcome ($y = 1$ if the exceedance occurred, else 0). Define the *calibration compliance*

$$c \;:=\; 1 - |q - y| \in [0,1].$$

If a peak ground acceleration A_{obs} is measured with standard uncertainty $\sigma_A > 0$, quantify *threshold ambiguity* by

$$u \;:=\; \exp\Big(-\tfrac{|A_{\text{obs}} - A_{\text{thr}}|}{\kappa\,\sigma_A}\Big) \;\in (0,1], \quad \kappa > 0,$$

so u is largest when the observation lies near the decision threshold. The *neutrosophic assessment* of the forecast is the triplet

$$T_{\text{haz}} := c(1-u), \qquad I_{\text{haz}} := u, \qquad F_{\text{haz}} := (1-c)(1-u),$$

which satisfies $T_{\text{haz}} + I_{\text{haz}} + F_{\text{haz}} = 1$. Here T_{haz} measures truth/achievement (well–calibrated forecast realized), F_{haz} measures falsity/failure (miscalibrated outcome), and I_{haz} captures ambiguity due to proximity to the threshold and sensor uncertainty.

Example 7.21 (Seismic Forecast: Neutrosophic Probabilistic Hazard Consistency)**.** At site s, a forecast states that peak ground acceleration will exceed $A_{\text{thr}} = 0.30\,g$ during a month with probability $q = 0.70$. The exceedance occurs ($y = 1$), so calibration compliance is $c = 1 - |q - y| = 0.70$. The sensor records $A_{\text{obs}} = 0.335\,g$ with standard uncertainty $\sigma_A = 0.05\,g$. Taking $\kappa = 1$, the threshold ambiguity is chosen at a natural half–information distance, $|A_{\text{obs}} - A_{\text{thr}}| = \sigma_A \ln 2$, thus

$$u = \exp\Big(-\frac{|A_{\text{obs}} - A_{\text{thr}}|}{\kappa\,\sigma_A}\Big) = \exp(-\ln 2) = \tfrac{1}{2}.$$

Hence the neutrosophic assessment triplet is

$$T_{\text{haz}} = c(1-u) = 0.70 \times 0.5 = 0.35, \quad I_{\text{haz}} = u = 0.50, \quad F_{\text{haz}} = (1-c)(1-u) = 0.30 \times 0.5 = 0.15,$$

which sums to 1. The forecast is broadly well calibrated (truth 0.35), ambiguity is high owing to proximity to the threshold (0.50), and falsity is low (0.15).

7.11 Neutrosophic Perceptual Sensitivity (Psychophysics)

Psychophysics quantifies relationships between physical stimuli and perceptual responses, estimating thresholds, scaling intensity, and modeling decision noise across sensory modalities [288–290]. Perceptual sensitivity measures an observer's ability to discriminate signal from noise, using d-prime, hit rates, false alarms, and psychometric functions [291–294]. Neutrosophic perceptual sensitivity characterizes discrimination performance via triplets—truth, indeterminacy, falsity—integrating accuracy, uncertainty, and conflict under variable, noisy, realistic experimental contexts.

Definition 7.22 (Neutrosophic Perceptual Sensitivity (Psychophysics))**.** Consider a yes/no detection experiment at stimulus intensity s_0 with n_S signal–present and n_N signal–absent trials. Let H and FA be the hit and false–alarm rates (use a log–linear correction if needed to avoid 0 or 1). Define

$$d' \;:=\; \Phi^{-1}(H) \;-\; \Phi^{-1}(FA), \qquad P_{\text{corr}} \;:=\; \Phi\big(d'/\sqrt{2}\big),$$

where Φ is the standard normal CDF. Approximate the standard error of d' by the delta method:

$$\sigma_{d'} \;\approx\; \sqrt{\frac{H(1-H)}{n_S\,\varphi(\Phi^{-1}(H))^2} \;+\; \frac{FA(1-FA)}{n_N\,\varphi(\Phi^{-1}(FA))^2}}\,,$$

with φ the standard normal PDF. Map uncertainty to $[0,1]$ via

$$u \;:=\; 1 - \exp(-\sigma_{d'}).$$

The *neutrosophic sensitivity* at s_0 is the triplet

$$T_{\text{perc}} := P_{\text{corr}}(1-u), \qquad I_{\text{perc}} := u, \qquad F_{\text{perc}} := (1 - P_{\text{corr}})(1-u),$$

so $T_{\text{perc}} + I_{\text{perc}} + F_{\text{perc}} = 1$. Thus T_{perc} grades achieved discriminability, F_{perc} grades failure to discriminate, and I_{perc} quantifies ambiguity from finite–sample and criterion variability.

Example 7.23 (Auditory Detection: Neutrosophic Perceptual Sensitivity)**.** An observer performs a yes/no tone–in–noise task at intensity s_0 with $n_S = n_N = 100$ trials, yielding hit rate $H = 0.76$ and false–alarm rate $FA = 0.24$. Using signal–detection theory,

$$d' = \Phi^{-1}(H) - \Phi^{-1}(FA) \approx 1.41, \qquad P_{\text{corr}} = \Phi\big(d'/\sqrt{2}\big) \approx 0.84.$$

By the delta method, with φ the standard normal pdf,

$$\sigma_{d'} \approx \sqrt{\frac{H(1-H)}{n_S\,\varphi(\Phi^{-1}(H))^2} + \frac{FA(1-FA)}{n_N\,\varphi(\Phi^{-1}(FA))^2}} \approx 0.195,$$

so $u = 1 - e^{-\sigma_{d'}} \approx 0.177$. Therefore

$$T_{\text{perc}} = P_{\text{corr}}(1-u) \approx 0.84 \times 0.823 \approx 0.693, \quad I_{\text{perc}} = u \approx 0.177, \quad F_{\text{perc}} = (1-P_{\text{corr}})(1-u) \approx 0.16 \times 0.823 \approx 0.131,$$

with small rounding error to 1. Thus performance shows strong discriminability (truth), modest ambiguity from sampling/criterion variability, and limited failure.

Chapter 8: Neutrosophic Chemistry

In this chapter, we explore and reflect on the concept of Neutrosophic Chemistry. In the field of Chemistry as well, Neutrosophic Sets have been applied in various ways. The necessary preliminary definitions are presented below.

Definition 8.1 (Neutrosophic Triplets and Mixers)**.** (cf. [295–297]) A *neutrosophic truth value* is a triplet $v = (T, I, F) \in [0,1]^3$, interpreted as degrees of truth/achievement (T), indeterminacy/ambiguity (I), and falsity/failure (F). When needed, we adopt the normalization $T + I + F \le 1$. Fix a left–continuous t–norm $t : [0,1]^2 \to [0,1]$ and its dual t–conorm $s : [0,1]^2 \to [0,1]$ (both associative, commutative, and monotone). We write

$$(T,I,F) \oplus (T',I',F') := \big(s(T,T'),\, s(I,I'),\, s(F,F')\big), \qquad (T,I,F) \otimes (T',I',F') := \big(t(T,T'),\, t(I,I'),\, t(F,F')\big).$$

Remark 8.2 (Residual–to–Degree Template)**.** Given a nonnegative *residual* $r \ge 0$ (how much a law/constraint is violated), an uncertainty mass $u \in [0,1]$, and a scale $\tau > 0$, define

$$c := e^{-r/\tau} \in (0,1], \qquad T := (1-u)\,c, \quad I := u, \quad F := (1-u)\,(1-c).$$

This generic template will be reused below to turn physical/chemical residuals into neutrosophic degrees (T, I, F).

8.1 Neutrosophic Chemical Equilibrium

Chemical Equilibrium describes conditions where forward and reverse reactions balance, minimizing free-energy residuals while acknowledging measurement uncertainty and model deviations [298–300]. Neutrosophic Chemical Equilibrium assigns a triplet grading how closely reaction conditions satisfy Gibbs equilibrium while isolating uncertainty and deviations.

Definition 8.3 (Neutrosophic Chemical Equilibrium)**.** Let a reaction $\sum_{i=1}^{m} \nu_i X_i \rightleftharpoons 0$ have chemical potentials μ_i (or activities a_i) at a fixed (T, P). The classical equilibrium condition is $\sum_i \nu_i \mu_i = 0$ (equivalently $Q = K$ in terms of reaction quotient Q and constant K). Define the equilibrium residual

$$r_{\mathrm{eq}} := \Big|\textstyle\sum_{i=1}^{m} \nu_i \mu_i\Big| \quad (\text{or } r_{\mathrm{eq}} := |\ln(Q/K)|).$$

Let $u_{\mathrm{eq}} \in [0,1]$ quantify uncertainty (finite statistics, sensor/model error), and $\tau_{\mathrm{eq}} > 0$ a scale. The *neutrosophic equilibrium degree* is

$$(T_{\mathrm{eq}}, I_{\mathrm{eq}}, F_{\mathrm{eq}}) := \Big((1-u_{\mathrm{eq}})\, e^{-r_{\mathrm{eq}}/\tau_{\mathrm{eq}}},\;\; u_{\mathrm{eq}},\;\; (1-u_{\mathrm{eq}})\big(1 - e^{-r_{\mathrm{eq}}/\tau_{\mathrm{eq}}}\big)\Big).$$

Example 8.4 (Industrial Ammonia Synthesis: Neutrosophic Chemical Equilibrium)**.** In a Haber–Bosch reactor at fixed (T, P), in situ spectroscopy yields $\ln(Q/K) = 0.20$ for $N_2 + 3H_2 \rightleftharpoons 2NH_3$, so $r_{\mathrm{eq}} = |\ln(Q/K)| = 0.20$. From repeatability and sensor modeling we set $u_{\mathrm{eq}} = 0.15$, and choose a scale $\tau_{\mathrm{eq}} = 0.50$. Then $\exp(-r_{\mathrm{eq}}/\tau_{\mathrm{eq}}) = e^{-0.40} \approx 0.6703$, hence

$$T_{\mathrm{eq}} = (1 - 0.15) \cdot 0.6703 \approx 0.57, \quad I_{\mathrm{eq}} = 0.15, \quad F_{\mathrm{eq}} = (1 - 0.15) \cdot (1 - 0.6703) \approx 0.28.$$

The process is close to Gibbs equilibrium with moderate residual and quantified uncertainty.

8.1.1 Neutrosophic Chemical Law

Chemical Law encodes conservation, rate, and equation constraints, evaluating compliance via residual magnitudes, uncertainty estimates, and penalized deviations from observations [301–303]. Neutrosophic Chemical Law maps residuals of conservation, rate, or equation constraints to truth, indeterminacy, and falsity degrees quantifying law compliance.

Definition 8.5 (Neutrosophic Chemical Law)**.** Let a *chemical law* be encoded as constraints $g_\ell(x) = 0$, $\ell = 1, \dots, L$, on a state vector x (e.g. mass/charge/atom conservation; rate laws; equation of state). Define the law residual $r_{\mathrm{law}} := \big(\sum_{\ell=1}^{L} w_\ell\, |g_\ell(x)|^p\big)^{1/p}$ for weights $w_\ell > 0$ and $p \in [1, \infty]$. With uncertainty $u_{\mathrm{law}} \in [0,1]$ and scale $\tau_{\mathrm{law}} > 0$, set

$$(T_{\mathrm{law}}, I_{\mathrm{law}}, F_{\mathrm{law}}) := \Big((1-u_{\mathrm{law}})\, e^{-r_{\mathrm{law}}/\tau_{\mathrm{law}}},\;\; u_{\mathrm{law}},\;\; (1-u_{\mathrm{law}})\big(1 - e^{-r_{\mathrm{law}}/\tau_{\mathrm{law}}}\big)\Big).$$

Example 8.6 (Leaky Batch Reactor: Neutrosophic Chemical Law)**.** A sealed batch esterification run is post-audited against mass conservation and a rate-law fit, encoded as constraints $g_\ell(x) = 0$. The aggregate residual is $r_{\mathrm{law}} = (\sum_\ell w_\ell |g_\ell|^2)^{1/2} = 0.05$ (in normalized units), with uncertainty $u_{\mathrm{law}} = 0.10$ from scale drift; take $\tau_{\mathrm{law}} = 0.10$. Then $e^{-r_{\mathrm{law}}/\tau_{\mathrm{law}}} = e^{-0.5} \approx 0.6065$ and

$$T_{\mathrm{law}} \approx 0.9 \cdot 0.6065 = 0.546, \quad I_{\mathrm{law}} = 0.10, \quad F_{\mathrm{law}} \approx 0.9 \cdot (1 - 0.6065) = 0.354.$$

Small but systematic deviations (a suspected micro–leak) reduce compliance while uncertainty is explicit.

8.2 Neutrosophic Atom

Atom denotes an identifiable elemental entity inferred from features like charge, mass, and configuration, with ambiguous or conflicting evidence considered [304–307]. Neutrosophic Atom labels candidate particles with element-wise triplets reflecting evidential support, conflicting features, and measurement-induced ambiguity across instruments.

Definition 8.7 (Neutrosophic Atom)**.** Let $\mathcal{E}$ be the set of element symbols and, for a particle/cluster y, let features $\varphi(y)$ include estimates of atomic number, isotopic mass, charge, and electronic configuration. A *neutrosophic element assignment* is a map

$$\Lambda_{\text{atom}} : \mathcal{E} \times \{\text{candidates } y\} \to [0,1]^3, \qquad (e, y) \mapsto \big(T_e(y), I_e(y), F_e(y)\big),$$

where $T_e(y)$ measures support that y is an atom of element e, $F_e(y)$ measures contradiction (incompatible features), and $I_e(y)$ captures indeterminacy (e.g. poor resolution, transient ionization). One may require $\sup_{e\in\mathcal{E}} T_e(y) \le 1$ and aggregate evidence via t/s–mixers over instruments.

Example 8.8 (Element Assignment in Noisy TOF–MS: Neutrosophic Atom)**.** A time–of–flight mass spectrum shows a peak at $m/z \approx 23$ with limited resolution. Comparing isotopic envelopes and charge states, the assignment map returns for y:

$$\Lambda_{\text{atom}}(\text{Na}, y) = (T, I, F) = (0.72,\ 0.20,\ 0.08),$$

reflecting strong support for Na^+, some ambiguity from overlapping fragment ions, and small contradiction from a weak alternative fit. Additional instruments (ICP–MS) may lower I via t/s–aggregation.

8.3 Neutrosophic Matter

Matter represents physical substance satisfying calibrated feature criteria such as density, neutrality, and spectra, while explicitly accounting for observational uncertainty [308–311]. Neutrosophic Matter evaluates a specimen's matter status by calibrated feature compliance, allocating residual uncertainty and assigning falsity to incompatible observations.

Definition 8.9 (Neutrosophic Matter)**.** For a specimen S with measurement stream $\mathcal{M}$, define a predicate "is ordinary matter" by a feature map $\psi(S;\mathcal{M})$ (mass density, charge neutrality, spectra). The *matter degree* is a triplet

$$(T_{\text{mat}}, I_{\text{mat}}, F_{\text{mat}}) := \big(\Theta^+(S),\ U(S),\ 1 - \Theta^+(S) - U(S)\big),$$

where $\Theta^+(S) \in [0,1]$ is a calibrated compliance to matter features, while $U(S) \in [0,1]$ encodes sensor/model uncertainty. Thus F_{mat} measures failure to meet matter criteria (e.g. fully ionized radiation field, pure vacuum within tolerance).

Example 8.10 (Aerosol Microdroplet: Neutrosophic Matter)**.** A laser–illuminated microdroplet stream is evaluated against the predicate "ordinary matter" using density, charge neutrality, and Raman signatures. Calibrated compliance yields $\Theta^+(S) = 0.80$; measurement/model uncertainty is $U(S) = 0.10$. Hence

$$(T_{\text{mat}}, I_{\text{mat}}, F_{\text{mat}}) = \big(0.80,\ 0.10,\ 0.10\big).$$

Occasional plasma glow near electrodes explains the small falsity component.

8.4 Neutrosophic Compound

Compound designates a specified stoichiometric and structural entity, assessed by compositional distance and structural conformity, with uncertainty and mismatches penalized [312–315]. Neutrosophic Compound grades compound identity by stoichiometric distance and structural conformity, propagating measurement uncertainty and penalizing mismatches through neutrosophic triplets.

Definition 8.11 (Neutrosophic Compound)**.** Let a *compound specification* be $(\mathbf{n}, \mathcal{S})$ where $\mathbf{n} = (n_1, \dots, n_m) \in \mathbb{N}^m$ is a stoichiometric vector over species set $\{X_1, \dots, X_m\}$, and $\mathcal{S}$ lists expected structural constraints (graph/geometry/fingerprint). For a sample with composition $\hat{\mathbf{c}}$ and structure score $\sigma \in [0,1]$, define a stoichiometric residual $r_{\text{st}} := \text{dist}\big(\hat{\mathbf{c}}, \lambda\mathbf{n}\big)$ minimized over $\lambda > 0$ (any norm). With uncertainty $u_{\text{cmp}} \in [0,1]$ and scale $\tau > 0$, the *compound degree* is

$$(T_{\text{cmp}}, I_{\text{cmp}}, F_{\text{cmp}}) := \Big((1 - u_{\text{cmp}})\, e^{-r_{\text{st}}/\tau}\, \sigma,\ \ u_{\text{cmp}},\ \ (1 - u_{\text{cmp}})\big(1 - e^{-r_{\text{st}}/\tau}\, \sigma\big)\Big).$$

Example 8.12 (Forensic Pill Identification (Acetaminophen): Neutrosophic Compound)**.** A seized tablet is compared with the specification $(\mathbf{n}, \mathcal{S})$ for acetaminophen. The composition distance to $\lambda\mathbf{n}$ is $r_{\text{st}} = 0.12$ (normalized), the structural score (NMR/IR/LC–MS fingerprint) is $\sigma = 0.90$, and uncertainty is $u_{\text{cmp}} = 0.10$ (sampling, excipients). With $\tau = 0.30$, $e^{-r_{\text{st}}/\tau} = e^{-0.4} \approx 0.6703$:

$$T_{\text{cmp}} \approx 0.9 \cdot 0.6703 \cdot 0.90 \approx 0.543, \quad I_{\text{cmp}} = 0.10, \quad F_{\text{cmp}} \approx 0.9 \cdot \big(1 - 0.6703 \cdot 0.90\big) \approx 0.357.$$

Evidence favors the target compound with quantified residual formulation mismatch.

8.5 Neutrosophic Mole

Mole denotes a quantity equal to Avogadro's number of entities, with closeness evaluated from counting residuals and associated estimation uncertainty [316–318]. Neutrosophic Mole quantifies closeness to one mole using counting residuals, encodes estimation uncertainty, and classifies failures when deviations become substantial.

Definition 8.13 (Neutrosophic Mole)**.** For a substance X and specimen containing N entities (with estimator $\widehat{N}$ and relative uncertainty $\eta \in [0,1]$), let N_A be Avogadro's constant. Define the residual $r_{\mathrm{mol}} := \left|\widehat{N}/N_A - 1\right|$. With scale $\tau_{\mathrm{mol}} > 0$ and uncertainty $u_{\mathrm{mol}} := \eta$, the *mole degree* is

$$(T_{\mathrm{mol}}, I_{\mathrm{mol}}, F_{\mathrm{mol}}) := \Big((1-u_{\mathrm{mol}})\, e^{-r_{\mathrm{mol}}/\tau_{\mathrm{mol}}},\ \ u_{\mathrm{mol}},\ \ (1-u_{\mathrm{mol}})\big(1-e^{-r_{\mathrm{mol}}/\tau_{\mathrm{mol}}}\big)\Big).$$

This grades how closely the specimen realizes "one mole" of X given counting and purity errors.

Example 8.14 (Preparing One Mole of NaCl: Neutrosophic Mole)**.** A lab prepares 1.00 L of nominal 1.00 M NaCl. Gravimetry/volumetry estimates the entity count $\widehat{N}$ with relative uncertainty $\eta = 0.08$; the relative deviation is $r_{\mathrm{mol}} = \left|\widehat{N}/N_A - 1\right| = 0.02$. With $\tau_{\mathrm{mol}} = 0.05$, $e^{-r_{\mathrm{mol}}/\tau_{\mathrm{mol}}} = e^{-0.4} \approx 0.6703$ and $u_{\mathrm{mol}} = \eta$:

$$T_{\mathrm{mol}} \approx 0.92 \cdot 0.6703 = 0.617, \quad I_{\mathrm{mol}} = 0.08, \quad F_{\mathrm{mol}} \approx 0.92 \cdot (1 - 0.6703) = 0.303.$$

The batch is acceptably close to one mole, with clear accounting of measurement variability.

8.6 Neutrosophic Phase

Phase specifies a thermodynamic state characterized by order parameters and free energies, allowing membership, incompatibility, and ambiguity across coexistence regions [319–321]. Neutrosophic Phase assigns phase membership, incompatibility, and ambiguity degrees from free-energy residuals, accommodating coexistence regions and near-critical transitional states.

Definition 8.15 (Neutrosophic Phase)**.** Let $\mathcal{P} = \{\alpha_1, \ldots, \alpha_k\}$ be candidate phases and x a micro/macro-state (e.g. order parameters, density). A *phase labeling* is a map

$$\Pi : \mathcal{P} \times \{x\} \to [0,1]^3, \qquad (\alpha, x) \mapsto \big(T_\alpha(x), I_\alpha(x), F_\alpha(x)\big),$$

where $T_\alpha(x)$ grades membership of x to phase α, $F_\alpha(x)$ grades incompatibility, and $I_\alpha(x)$ captures coexistence/transition uncertainties. A common construction uses free energies $G_\alpha(x)$ and $r_\alpha(x) := G_\alpha(x) - \min_\beta G_\beta(x)$ with the residual–to–degree template.

Example 8.16 (Water Near the Triple Point: Neutrosophic Phase)**.** At $T = 0.01^\circ$C and $P = 611$ Pa, free–energy estimates yield residuals $r_{\mathrm{liq}} = 0.01$, $r_{\mathrm{ice}} = 0.02$, $r_{\mathrm{vap}} = 0.45$ (arbitrary units), with uncertainty $u = 0.12$ and scale $\tau = 0.05$. Then $e^{-r/\tau}$ gives $\{0.8187, 0.6703, \approx 0.00012\}$, so

$$\begin{aligned}
(T_{\mathrm{liq}}, I_{\mathrm{liq}}, F_{\mathrm{liq}}) &\approx (0.720,\ 0.120,\ 0.160),\\
(T_{\mathrm{ice}}, I_{\mathrm{ice}}, F_{\mathrm{ice}}) &\approx (0.590,\ 0.120,\ 0.290),\\
(T_{\mathrm{vap}}, I_{\mathrm{vap}}, F_{\mathrm{vap}}) &\approx (0.0001,\ 0.120,\ 0.880).
\end{aligned}$$

Liquid and ice both have substantial membership, capturing phase coexistence; vapor is strongly disfavored.

8.7 Neutrosophic Energy

Energy quantifies a system's capacity for work or heat, with discrepancies between observed and theoretical values guiding confidence and uncertainty [322–324]. Neutrosophic Energy compares observed and theoretical energies, maps discrepancies to truth and falsity, and reserves indeterminacy for measurement or model uncertainty.

Definition 8.17 (Neutrosophic Energy)**.** For a system state x, let $E_{\mathrm{th}}(x)$ be a theoretical energy and $E_{\mathrm{obs}}(x)$ an estimator with standard uncertainty $\sigma_E > 0$. Set $r_E := |E_{\mathrm{obs}}(x) - E_{\mathrm{th}}(x)|$ and $u_E := 1 - e^{-\sigma_E/\kappa}$ for some $\kappa > 0$. The *neutrosophic energy consistency* is

$$(T_E, I_E, F_E) := \Big((1-u_E)\, e^{-r_E/\tau_E},\ \ u_E,\ \ (1-u_E)\big(1-e^{-r_E/\tau_E}\big)\Big),$$

with scale $\tau_E > 0$. Large T_E indicates agreement with theory; large F_E indicates systematic departures; I_E captures measurement/model ambiguity.

Example 8.18 (Battery Formation Enthalpy: Neutrosophic Energy)**.** A cathode reaction has $E_{\mathrm{th}} = -250$ kJ/mol; calorimetry gives $E_{\mathrm{obs}} = -240 \pm 8$ kJ/mol. Thus $r_E = |-240-(-250)| = 10$ and, with $\kappa = 20$, $u_E = 1 - e^{-8/20} = 1 - e^{-0.4} \approx 0.3297$. With $\tau_E = 20$, $e^{-r_E/\tau_E} = e^{-0.5} \approx 0.6065$:

$$T_E \approx (1 - 0.3297) \cdot 0.6065 = 0.406, \quad I_E \approx 0.330, \quad F_E \approx (1 - 0.3297) \cdot (1 - 0.6065) = 0.263.$$

Measurements moderately support the theoretical energy, with nontrivial uncertainty and limited falsity.

8.8 Neutrosophic Chemical Graph

A *chemical graph* represents a molecule as a labeled graph in which vertices denote atoms and edges denote pairwise chemical bonds [325, 326]. A neutrosophic chemical graph models molecules as graphs whose atoms and bonds carry neutrosophic triplets representing uncertainty and connectivity ambiguity. Related formalisms include the *fuzzy chemical graph* [327–331], in which memberships of atoms/bonds are graded in $[0, 1]$, and the *chemical hypergraph* [332–335], which uses hyperedges to represent multi-center interactions (e.g., delocalized or multi-atom bonding). Furthermore, since extensions of HyperGraphs such as SuperHyperGraphs [336–338] are already known, future research on Chemical SuperHyperGraphs and Molecular SuperHyperGraphs can be expected [339–341].

Definition 8.19 (Neutrosophic Chemical Graph)**.** [327] Let $\mathcal{A}$ be a finite set of *atom types* (e.g., H, C, N, O, ...) and $\mathcal{B}$ a finite set of *bond types* (e.g., single, double, aromatic). A *neutrosophic chemical graph* is a quadruple

$$\mathsf{G} = (V, E, \lambda_V, \lambda_E),$$

where $G = (V, E)$ is a simple (undirected) graph and

$$\lambda_V : V \times \mathcal{A} \to [0, 1]^3, \qquad \lambda_E : E \times \mathcal{B} \to [0, 1]^3$$

are *neutrosophic label maps* that assign to each vertex–type pair (v, a) and edge–type pair (e, b) a triplet (T, I, F) of degrees of truth/presence (T), indeterminacy/ambiguity (I), and falsity/absence (F). For a fixed $a \in \mathcal{A}$, $\lambda_V(v, a) = (T_{v,a}, I_{v,a}, F_{v,a})$ encodes the evidential support that v is of atom type a; similarly, for $b \in \mathcal{B}$, $\lambda_E(e, b) = (T_{e,b}, I_{e,b}, F_{e,b})$ encodes that e is a bond of type b.

Fix a left–continuous t–norm t and its dual t–conorm s. For an edge $e = \{u, v\}$, the *aggregate bond degrees* are

$$T_E(e) := \sup_{b \in \mathcal{B}} T_{e,b}, \qquad I_E(e) := \sup_{b \in \mathcal{B}} I_{e,b}, \qquad F_E(e) := \inf_{b \in \mathcal{B}} F_{e,b}.$$

For a walk $P = v_0 - v_1 - \cdots - v_k$ with edges $e_i = \{v_{i-1}, v_i\}$, define its *neutrosophic connectivity degrees* by

$$T(P) := t\big(T_E(e_1), \ldots, T_E(e_k)\big), \quad I(P) := s\big(I_E(e_1), \ldots, I_E(e_k)\big), \quad F(P) := s\big(F_E(e_1), \ldots, F_E(e_k)\big).$$

The *neutrosophic reachability* from u to v is obtained by taking the t–supremum over all $u \rightsquigarrow v$ walks:

$$T^*(u, v) := \sup_{P : u \rightsquigarrow v} T(P), \quad I^*(u, v) := \inf_{P : u \rightsquigarrow v} I(P), \quad F^*(u, v) := \inf_{P : u \rightsquigarrow v} F(P).$$

When all labels are crisp ($T \in \{0, 1\}$, $I = 0$, $F = 1 - T$), G reduces to an ordinary typed chemical graph.

Example 8.20 (Keto-Enol Tautomerism as a Neutrosophic Chemical Graph)**.** Let $\mathcal{A} = \{\mathrm{C}, \mathrm{O}, \mathrm{H}\}$ and $\mathcal{B} = \{\text{single}, \text{double}, \text{H-bond}\}$. Consider acetylacetone (pentane-2,4-dione) in solution, which rapidly interconverts between keto and enol forms. Take the graph $G = (V, E)$ with

$$V = \{C_1, C_2, C_3, C_4, C_5, O_1, O_2, H^*\}, \quad E = \big\{\{C_2, O_1\}, \{C_4, O_2\}, \{O_1, H^*\}, \{C_i, C_{i+1}\}_{i=1}^{4}\big\}.$$

Choose $t = \min$ and $s = \max$. Neutrosophic vertex labels (nonzero entries shown) encode near-certain atom typing:

$$\lambda_V(O_j, \mathrm{O}) = (0.98, 0.02, 0.00), \ \ \lambda_V(C_i, \mathrm{C}) = (0.99, 0.01, 0.00), \ \ \lambda_V(H^*, \mathrm{H}) = (0.97, 0.03, 0.00).$$

Key edge labels reflect tautomeric ambiguity (triplets are (T, I, F)):

$$\begin{aligned}
&\lambda_E(\{C_2, O_1\}, \text{double}) = (0.55, 0.35, 0.10), \quad \lambda_E(\{C_2, O_1\}, \text{single}) = (0.35, 0.50, 0.15),\\
&\lambda_E(\{C_4, O_2\}, \text{double}) = (0.70, 0.25, 0.05), \quad \lambda_E(\{C_4, O_2\}, \text{single}) = (0.20, 0.65, 0.15),\\
&\lambda_E(\{O_1, H^*\}, \text{single}) = (0.60, 0.30, 0.10), \quad \lambda_E(\{C_i, C_{i+1}\}, \text{single}) = (0.95, 0.04, 0.01) \ (i = 1, \ldots, 4).
\end{aligned}$$

Aggregate bond degrees (Definition: $T_E = \sup_b T_{e,b}$, $I_E = \sup_b I_{e,b}$, $F_E = \inf_b F_{e,b}$) give

$$\begin{aligned}
&T_E(C_2 - O_1) = 0.55, \ I_E = 0.50, \ F_E = 0.10; \quad T_E(C_4 - O_2) = 0.70, \ I_E = 0.65, \ F_E = 0.05;\\
&T_E(O_1 - H^*) = 0.60, \ I_E = 0.30, \ F_E = 0.10; \quad T_E(C_i - C_{i+1}) = 0.95, \ I_E = 0.04, \ F_E = 0.01.
\end{aligned}$$

For the path $P : C_1 \to C_2 \to O_1 \to H^*$, the neutrosophic connectivity is

$$T(P) = \min(0.95, 0.55, 0.60) = 0.55, \quad I(P) = \max(0.04, 0.50, 0.30) = 0.50, \quad F(P) = \max(0.01, 0.10, 0.10) = 0.10.$$

Interpretation. The C_2=O_1 linkage is partly double and partly single due to keto–enol dynamics, so reachability from C_1 to the enolic proton H^* carries moderate truth (0.55), high indeterminacy (0.50) from rapid tautomerization, and low falsity (0.10).

Example 8.21 (Aromatic Bonding in Benzene with Measurement Ambiguity)**.** Let $\mathcal{A} = \{\mathrm{C}, \mathrm{H}\}$ and $\mathcal{B} = \{\text{single}, \text{double}, \text{aromatic}\}$. Consider benzene C_6H_6 with vertex set $V = \{C_1, \ldots, C_6, H_1, \ldots, H_6\}$ and edges $E = \{\{C_i, C_{i+1}\}\ (i \bmod 6),\ \{C_i, H_i\}\ (i = 1, \ldots, 6)\}$. Choose $t = \min$ and $s = \max$. High-confidence atom typing:

$$\lambda_V(C_i, \mathrm{C}) = (0.99, 0.01, 0.00), \qquad \lambda_V(H_i, \mathrm{H}) = (0.99, 0.01, 0.00).$$

Neutrosophic edge labels incorporate spectroscopic/microscopic evidence (STM/NMR) and noise: for each ring edge $e_i = \{C_i, C_{i+1}\}$,

$$\lambda_E(e_i, \text{aromatic}) = (0.90, 0.08, 0.02), \quad \lambda_E(e_i, \text{single}) = (0.15, 0.20, 0.70), \quad \lambda_E(e_i, \text{double}) = (0.15, 0.20, 0.70).$$

For each C–H edge $f_i = \{C_i, H_i\}$,

$$\lambda_E(f_i, \text{single}) = (0.98, 0.02, 0.00).$$

Aggregate bond degrees yield (for all ring edges) $T_E = 0.90$, $I_E = 0.20$, $F_E = 0.02$ (since $I_E = \max\{0.08, 0.20, 0.20\} = 0.20$), and for C–H edges $T_E = 0.98$, $I_E = 0.02$, $F_E = 0.00$. For the three-edge path $P : C_1 \to C_2 \to C_3 \to C_4$,

$$T(P) = \min(0.90, 0.90, 0.90) = 0.90, \quad I(P) = \max(0.20, 0.20, 0.20) = 0.20, \quad F(P) = \max(0.02, 0.02, 0.02) = 0.02.$$

Interpretation. The ring bonds are predominantly aromatic (high T), but experimental uncertainty and model idealizations contribute a non-negligible $I = 0.20$; conflicting single/double assignments are largely falsified ($F = 0.02$) for benzene under these conditions.

8.9 Neutrosophic Chemical Reaction

A chemical reaction is a process where reactants transform into products through bond breaking and forming, releasing or absorbing energy [342–344]. A neutrosophic chemical reaction assigns neutrosophic degrees to reactants, products, and conditions, quantifying feasibility, uncertainty, and failure, with guards, kinetics.

Definition 8.22 (Neutrosophic Chemical Reaction)**.** Fix species S and conditions Θ. A (formal) reaction is a pair $(\alpha, \beta) \in \mathbb{N}^{(S)} \times \mathbb{N}^{(S)}$, written

$$\alpha \longrightarrow \beta, \quad \text{where} \quad \alpha = \sum_{x \in S} \alpha(x)\, x, \ \ \beta = \sum_{x \in S} \beta(x)\, x.$$

Let $M \in \mathbb{Z}_{\geq 0}^{E \times S}$ be the element–species incidence matrix (rows = elements, columns = species). The reaction is *balanced* iff $M\alpha = M\beta$.

A *neutrosophic chemical reaction* is a quadruple

$$\mathsf{R} = (\alpha, \beta, \theta, \rho),$$

with $\theta \in \Theta$ and

$$\rho : \Theta \longrightarrow [0,1]^3, \qquad \theta \longmapsto \big(T_{\mathsf{R}}(\theta),\ I_{\mathsf{R}}(\theta),\ F_{\mathsf{R}}(\theta)\big),$$

such that:

1. *Guarded feasibility:* if $M\alpha \neq M\beta$, then $F_{\mathsf{R}}(\theta) = 1$ and $T_{\mathsf{R}}(\theta) = 0$ for all θ.
2. *Condition monotonicity:* improving a required condition (e.g., raising temperature within a favorable range or adding a catalyst) does not decrease T_{R} nor increase F_{R}.
3. *Mass–action interface (optional kinetics):* given concentrations $c \in \mathbb{R}_{\geq 0}^{S}$, one may define a *neutrosophic propensity triplet*
$$\Pi_{\mathsf{R}}(c, \theta) := \Big(k\, T_{\mathsf{R}}(\theta) \prod_{x \in S} c(x)^{\alpha(x)},\ \ u(\theta),\ \ k'\, F_{\mathsf{R}}(\theta) \prod_{x \in S} c(x)^{\beta(x)} \Big),$$
where $k, k' \geq 0$ are nominal forward/backward rate scales and $u(\theta) \in [0,1]$ summarizes uncertainty (e.g., unknown mechanism). This couples the neutrosophic evaluation with standard kinetics if desired.

In short, $(T_{\mathsf{R}}, I_{\mathsf{R}}, F_{\mathsf{R}})$ quantify, under θ, the degrees to which the balanced transformation $\alpha \to \beta$ is realized (truth), remains ambiguous (indeterminacy), or fails/is inhibited (falsity).

Example 8.23 (Esterification with water removal (Fischer esterification))**.** *Species and stoichiometry.* Let $S = \{\mathrm{Ac}, \mathrm{EtOH}, \mathrm{EtOAc}, \mathrm{H_2O}\}$ denote acetic acid, ethanol, ethyl acetate, and water. The formal reaction is

$$\alpha = \mathrm{Ac} + \mathrm{EtOH} \longrightarrow \beta = \mathrm{EtOAc} + \mathrm{H_2O}.$$

With elements $\{\mathrm{C}, \mathrm{H}, \mathrm{O}\}$ and column order $(\mathrm{Ac}, \mathrm{EtOH}, \mathrm{EtOAc}, \mathrm{H_2O})$, the element–species incidence matrix is

$$M = \begin{pmatrix} 2 & 2 & 4 & 0 \\ 4 & 6 & 8 & 2 \\ 2 & 1 & 2 & 1 \end{pmatrix}, \qquad M\alpha = (4, 10, 3)^{\top} = M\beta,$$

so the reaction is balanced.

Conditions and neutrosophic degrees. Take θ: toluene as azeotroping solvent, 60°C, 5 mol% $\mathrm{H_2SO_4}$, Dean–Stark water removal, reflux 4 h. Based on equilibrium data and practical conversion with continuous water removal, assign

$$\rho(\theta) = \big(T_{\mathsf{R}}(\theta), I_{\mathsf{R}}(\theta), F_{\mathsf{R}}(\theta)\big) = (0.85,\ 0.10,\ 0.05).$$

Truth is high (favorable equilibrium and Le Châtelier shift), indeterminacy moderate (titration/SN uncertainty), falsity low (side hydrolysis minimal).

Optional mass–action interface. With (normalized) concentrations $c(\mathrm{Ac}) = 1.0$, $c(\mathrm{EtOH}) = 1.2$, $c(\mathrm{EtOAc}) = 0.1$, $c(\mathrm{H_2O}) = 0.05$, and rate scales $k = 1.2$, $k' = 0.15$, the neutrosophic propensity triplet is

$$\Pi_{\mathsf{R}}(c, \theta) = \Big(\underbrace{k\, T_{\mathsf{R}}\, c(\mathrm{Ac})\, c(\mathrm{EtOH})}_{\text{forward}},\ \underbrace{I_{\mathsf{R}}}_{\text{uncertainty}},\ \underbrace{k'\, F_{\mathsf{R}}\, c(\mathrm{EtOAc})\, c(\mathrm{H_2O})}_{\text{inhibition/backward}}\Big) = (1.224,\ 0.10,\ 0.00075).$$

Example 8.24 (Ammonia synthesis (Haber–Bosch) under ambient vs. industrial conditions)**.** *Species and stoichiometry.* Let $S = \{\mathrm{N_2}, \mathrm{H_2}, \mathrm{NH_3}\}$ and

$$\alpha = \mathrm{N_2} + 3\,\mathrm{H_2} \longrightarrow \beta = 2\,\mathrm{NH_3}.$$

With elements $\{\mathrm{N}, \mathrm{H}\}$ and columns $(\mathrm{N_2}, \mathrm{H_2}, \mathrm{NH_3})$,

$$M = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 3 \end{pmatrix}, \qquad M\alpha = (2, 6)^{\top} = M\beta,$$

so the reaction is balanced.

Ambient conditions (unfavorable). Let θ_{amb}: 25°C, 1 bar, no catalyst. Assign

$$\rho(\theta_{\mathrm{amb}}) = (T_{\mathsf{R}}, I_{\mathsf{R}}, F_{\mathsf{R}}) = (0.02,\ 0.18,\ 0.80),$$

reflecting strong thermodynamic/kinetic inhibition at ambient conditions and some indeterminacy from trace catalytic surfaces/measurement noise. With nominal $k = 10^{-6}$, $k' = 10^{-2}$ and (normalized) $c(\mathrm{N_2}) = 0.9$, $c(\mathrm{H_2}) = 2.7$, $c(\mathrm{NH_3}) = 0.0$, the propensity is

$$\Pi_{\mathsf{R}}(c, \theta_{\mathrm{amb}}) = \Big(10^{-6}\cdot 0.02\cdot 0.9\cdot (2.7)^3,\ 0.18,\ 10^{-2}\cdot 0.80\cdot 0\Big) \approx (3.5\times 10^{-7},\ 0.18,\ 0).$$

Industrial conditions (favorable). Let θ_{ind}: 450°C, 150 bar, promoted Fe catalyst, recycle with continuous $\mathrm{NH_3}$ removal. Assign

$$\rho(\theta_{\mathrm{ind}}) = (0.85,\ 0.10,\ 0.05),$$

capturing high feasibility with modest uncertainty (model/data spread) and low falsity (occasional deactivation). With $k = 0.08$, $k' = 0.02$ and $c(\mathrm{N_2}) = 0.7$, $c(\mathrm{H_2}) = 2.1$, $c(\mathrm{NH_3}) = 0.4$,

$$\Pi_{\mathsf{R}}(c, \theta_{\mathrm{ind}}) = \Big(0.08\cdot 0.85\cdot 0.7\cdot (2.1)^3,\ 0.10,\ 0.02\cdot 0.05\cdot (0.4)^2\Big) \approx (0.55,\ 0.10,\ 0.00016).$$

The guarded feasibility and condition monotonicity are respected: moving from θ_{amb} to θ_{ind} increases T_{R} and reduces F_{R}.

8.10 Neutrosophic Stoichiometric Compliance

NSC quantifies stoichiometric balance using normalized mass residual, partitioning evidence into truth, uncertainty, and structured imbalance from measurements.

Definition 8.25 (Neutrosophic Stoichiometric Compliance (NSC))**.** Let a (possibly overall) reaction be encoded by a stoichiometric vector $\nu = (\nu_1, \ldots, \nu_m) \in \mathbb{Z}^m$ over species $\{1, \ldots, m\}$, with $\nu_i < 0$ for reactants and $\nu_i > 0$ for products. Let $M_i > 0$ be molar masses and $c_i \geq 0$ measured concentrations (or amounts n_i). Define the mass-balance residual

$$R \;:=\; \frac{\left|\sum_{i=1}^{m} \nu_i \, M_i \, c_i\right|}{\sum_{i=1}^{m} M_i \, c_i + \varepsilon} \;\in [0, \infty),$$

with a small $\varepsilon > 0$ to avoid division by zero. Let $u \in [0, 1]$ quantify measurement/model uncertainty (e.g., a pooled coefficient of variation or confidence deficit), and fix a scale $\tau > 0$. The *neutrosophic stoichiometric compliance* is the triplet

$$T_{\mathrm{nsc}} \;:=\; e^{-R/\tau}\,(1-u), \qquad I_{\mathrm{nsc}} \;:=\; u, \qquad F_{\mathrm{nsc}} \;:=\; \left(1 - e^{-R/\tau}\right)(1-u).$$

Then $T_{\mathrm{nsc}} + I_{\mathrm{nsc}} + F_{\mathrm{nsc}} = 1$, where T_{nsc} grades how well the measurements satisfy the stoichiometric balance, F_{nsc} grades structured imbalance, and I_{nsc} captures uncertainty.

Example 8.26 (NSC: Combustion bench test with slight mass–balance drift)**.** Consider $\mathrm{CH_4} + 2\,\mathrm{O_2} \to \mathrm{CO_2} + 2\,\mathrm{H_2O}$ with molar masses $M(\mathrm{CH_4}) = 16$, $M(\mathrm{O_2}) = 32$, $M(\mathrm{CO_2}) = 44$, $M(\mathrm{H_2O}) = 18$ (g/mol). Suppose measured (dimensionless) amounts are $c_{\mathrm{CH_4}} = 1.00$, $c_{\mathrm{O_2}} = 1.95$, $c_{\mathrm{CO_2}} = 0.98$, $c_{\mathrm{H_2O}} = 1.90$. With stoichiometric vector $\nu = (-1, -2, +1, +2)$,

$$R = \frac{\left|-16(1.00) - 2 \cdot 32(1.95) + 44(0.98) + 2 \cdot 18(1.90)\right|}{16(1.00) + 32(1.95) + 44(0.98) + 18(1.90)} = \frac{29.28}{155.72} \approx 0.188.$$

Take uncertainty $u = 0.10$ and scale $\tau = 0.20$. Then

$$T_{\mathrm{nsc}} = e^{-0.188/0.20}(1 - 0.10) \approx 0.352, \quad I_{\mathrm{nsc}} = 0.10, \quad F_{\mathrm{nsc}} = \left(1 - e^{-0.188/0.20}\right)(1 - 0.10) \approx 0.548.$$

Thus the run shows moderate imbalance (large F_{nsc}) beyond the modest measurement uncertainty.

8.11 Neutrosophic Spectral Assignment Confidence

NSAC evaluates spectral assignment by cosine similarity and an ambiguity index, yielding triplets for presence, indeterminacy, and absence of functional groups.

Definition 8.27 (Neutrosophic Spectral Assignment Confidence (NSAC))**.** Let $\mathcal{S}$ be an acquired spectrum (e.g., NMR/IR/MS) represented by a feature vector $\varphi(\mathcal{S}) \in \mathbb{R}^d_{\geq 0}$ (normalized to unit ℓ_2-norm). For a candidate functional group or substructure g with predicted feature vector $\varphi(g) \in \mathbb{R}^d_{\geq 0}$ (also unit-normalized), define the similarity

$$\mathrm{sim}(g \mid \mathcal{S}) \;:=\; \tfrac{1}{2}\big(1 + \langle \varphi(\mathcal{S}), \varphi(g) \rangle\big) \;\in [0, 1].$$

Let $\eta \in [0, 1]$ measure peak overlap/noise (e.g., $\eta = \exp(-\mathrm{SNR}/\kappa)$ or a resolution-based ambiguity index), and set $u := \max\{0, \min\{1, \eta\}\}$. The *neutrosophic confidence* that g is present in $\mathcal{S}$ is

$$T_{\mathrm{nsac}}(g \mid \mathcal{S}) \;:=\; \mathrm{sim}(g \mid \mathcal{S})\,(1-u), \qquad I_{\mathrm{nsac}}(g \mid \mathcal{S}) \;:=\; u, \qquad F_{\mathrm{nsac}}(g \mid \mathcal{S}) \;:=\; \big(1 - \mathrm{sim}(g \mid \mathcal{S})\big)\,(1-u).$$

Thus $T + I + F = 1$; high similarity and low ambiguity yield large T_{nsac}, while ambiguity raises I_{nsac} and mismatch raises F_{nsac}. Assignments over a set of candidates $\mathcal{G}$ can be aggregated by any monotone t–norm/t–conorm mixer on the triplets.

Example 8.28 (NSAC: IR/NMR assignment of a carbonyl group under peak overlap)**.** Let $\mathcal{S}$ be an IR/NMR spectrum and $g = \mathrm{C{=}O}$ a candidate group. Assume unit–normalized feature vectors yield $\mathrm{sim}(g \mid \mathcal{S}) = \frac{1}{2}(1 + \langle \varphi(\mathcal{S}), \varphi(g) \rangle) = 0.82$. Suppose peak crowding/noise gives an ambiguity index $\eta = 0.25$ (so $u = 0.25$). Then the neutrosophic confidence is

$$T_{\mathrm{nsac}} = 0.82(1 - 0.25) = 0.615, \quad I_{\mathrm{nsac}} = 0.25, \quad F_{\mathrm{nsac}} = (1 - 0.82)(1 - 0.25) = 0.135.$$

Hence the carbonyl assignment is strongly supported (T_{nsac} high), with some indeterminacy from overlapping peaks.

8.12 Neutrosophic Yield–Selectivity Profile

NYSP aggregates yield and selectivity via a t-norm, discounting by uncertainty, to grade process success, ambiguity, and failure.

Definition 8.29 (Neutrosophic Yield–Selectivity Profile (NYSP))**.** Consider a target product P^* formed from a limiting reactant with theoretical amount $n_{\max} > 0$. Let $n_{P^*} \geq 0$ be the measured amount of P^*, $n_{\text{conv}} \geq 0$ the consumed amount of the limiting reactant, and define *fractional yield* and *selectivity*

$$Y := \frac{n_{P^*}}{n_{\max}} \in [0,1], \qquad S := \frac{n_{P^*}}{n_{\text{conv}}} \in [0,1] \ \ (\text{set } S = 0 \text{ if } n_{\text{conv}} = 0).$$

Fix a continuous t–norm $t : [0,1]^2 \to [0,1]$ (e.g., product or minimum) and let $u \in [0,1]$ summarize uncertainty from analytics/normalization (e.g., GC response factors, mass balance closure). The *neutrosophic yield–selectivity profile* of the run is

$$T_{\text{nys}} := t(Y,S)\,(1-u), \qquad I_{\text{nys}} := u, \qquad F_{\text{nys}} := \big(1 - t(Y,S)\big)\,(1-u).$$

Hence T_{nys} rewards *simultaneously* high yield and selectivity (via t), F_{nys} penalizes their deficit, and I_{nys} captures measurement/model ambiguity. Across multiple runs $\{r\}$, triplets can be aggregated by t–norm/t–conorm or reliability-weighted means.

Example 8.30 (NYSP: Batch synthesis with decent yield and good selectivity)**.** A target product P^* has theoretical amount $n_{\max} = 100\,\text{mmol}$. The batch delivers $n_{P^*} = 72\,\text{mmol}$ while the limiting reactant consumed is $n_{\text{conv}} = 90\,\text{mmol}$. Then

$$Y = \frac{72}{100} = 0.72, \qquad S = \frac{72}{90} = 0.80.$$

Using the product t–norm $t(Y,S) = YS = 0.576$ and uncertainty $u = 0.10$,

$$T_{\text{nys}} = 0.576(1-0.10) = 0.518, \quad I_{\text{nys}} = 0.10, \quad F_{\text{nys}} = \big(1-0.576\big)(1-0.10) = 0.382.$$

The run achieves simultaneous moderate yield and selectivity (reflected in T_{nys}), with limited ambiguity and a residual shortfall from the ideal.

Chapter 9: Neutrosophic Philosophy (Neutrosophy)

Neutrosophic Philosophy has attracted extensive research from multiple perspectives. In this section, we review key concepts—such as Neutrosophic Paradoxes—and related developments (cf. [41, 345–349]).

9.1 Neutrosophic Paradoxes

Paradoxes are statements or situations producing self-contradiction or unsolvable cycles, exposing hidden assumptions and limits of classical logic and consistency. Neutrosophic paradoxes treat contradictions using truth, indeterminacy, falsity triplets, modeling partial validity, simultaneous opposites, and context-dependent resolutions within uncertain reasoning [350–352].

Definition 9.1 (Neutrosophic Paradox and Degree of Paradoxicity)**.** [350–352] Given a neutrosophic truth value $v = (T, I, F)$, its *degree of paradoxicity* is any monotone measure of simultaneous truth and falsity, e.g.

$$\pi_{\min}(v) := \min\{T, F\}, \qquad \pi_{\times}(v) := T \cdot F.$$

A statement (or state) is a *neutrosophic paradox* when $\pi(v) > 0$ for a chosen $\pi \in \{\pi_{\min}, \pi_{\times}, \ldots\}$; that is, it carries concurrent (nonzero) degrees of truth and falsity, possibly with nonzero indeterminacy I. This formalizes paradoxical or mixed-status phenomena while preserving graded assessment.

Example 9.2 (Neutrosophic Paradoxicity in Autonomous-Driving Hazard Detection)**.** An autonomous vehicle must decide whether a partially occluded object is a *pedestrian now entering the crosswalk*. Two perception modules yield conflicting evidence: the camera classifier fires with confidence 0.70 (supporting "pedestrian entering"), while LiDAR tracking suggests "static object" with confidence 0.50; heavy rain and occlusion add ambiguity of 0.20. Model the statement S: "A pedestrian is entering *now*" by the neutrosophic triplet

$$v(S) = (T, I, F) = (0.70,\ 0.20,\ 0.50).$$

The degree of paradoxicity (simultaneous truth and falsity) can be quantified, for instance, by

$$\pi_{\min}(v) = \min\{T, F\} = 0.50, \qquad \pi_{\times}(v) = T \cdot F = 0.35.$$

Hence S is a *neutrosophic paradox*: both "true" (camera cues) and "false" (LiDAR cues) to substantial, concurrent degrees, while $I = 0.20$ captures residual uncertainty from adverse conditions. In a safety controller, such paradoxicity can trigger a conservative action (e.g., slow/stop) without collapsing the conflicting evidence to a binary verdict.

Chapter 10: Other Uncertain Concepts

In this section, we examine other uncertain concepts and discuss their roles and implications.

10.1 Neutrosophic Successful System

Evaluates systems against goals using triplet degrees of success, indeterminacy, and failure, aggregating across goals and time to determine acceptability [353–355].

Definition 10.1 (Neutrosophic Successful System)**.** Let $\mathcal{S}$ be a (possibly dynamic) system and let $\mathcal{G}$ be a nonempty set of goals. A *neutrosophic evaluation* of $\mathcal{S}$ (at a configuration/state x) is a map

$$\varepsilon_x : \mathcal{G} \longrightarrow [0,1]^3, \qquad g \longmapsto \varepsilon_x(g) = (T_x(g), I_x(g), F_x(g)),$$

where $T_x(g)$, $I_x(g)$, $F_x(g)$ denote, respectively, the degrees of success (truth), indeterminacy, and failure (falsity) of $\mathcal{S}$ with respect to goal g. Intuitively, a neutrosophic successful system may succeed on some goals while being only partially successful or even failing on others, in accordance with the neutrosophic paradigm.

To aggregate goal–wise judgments, fix a *goal aggregator*

$$\mathsf{A} : \big([0,1]^3\big)^{\mathcal{G}} \longrightarrow [0,1]^3, \qquad \mathsf{A}\big((T(g), I(g), F(g))_{g\in\mathcal{G}}\big) = (T^*, I^*, F^*),$$

which is monotone increasing in each $T(g)$ and monotone decreasing in each $I(g)$ and $F(g)$. Given thresholds $\boldsymbol{\theta} = (\theta_T, \theta_I, \theta_F) \in [0,1]^3$, we say that $\mathcal{S}$ is $\boldsymbol{\theta}$*–neutrosophically successful at* x if

$$T^* \geq \theta_T, \qquad I^* \leq \theta_I, \qquad F^* \leq \theta_F.$$

When $\mathcal{S}$ is time–dependent, write ε_t and define success on a time horizon $H \subseteq \mathbb{R}_{\geq 0}$ by composing A with any time–aggregator A_{time} (e.g. integral/mean or a t–norm/t–conorm based mixer).

In a neutrosophic system, the usual attributes of classical systems (adaptivity, self-organization, etc.) are themselves assessed by neutrosophic triplets (t, i, f); the present definition allows such (t, i, f)–attributes to be encoded goal–wise via ε_x.

Example 10.2 (Hospital Emergency Department)**.** Let $\mathcal{S}$ be an emergency department (ED, [356]) and let the goal set be $\mathcal{G} = \{g_1, g_2, g_3\}$ with weights $w_{g_1} = 0.5$, $w_{g_2} = 0.3$, $w_{g_3} = 0.2$ (timeliness is most critical). We evaluate the ED on a given day x by neutrosophic triplets $(T_x(g), I_x(g), F_x(g)) \in [0,1]^3$ as follows:

goal g	w_g	$T_x(g)$	$I_x(g)$	$F_x(g)$
g_1 = "treat within 4h"	0.5	0.75	0.10	0.15
g_2 = "patient safety"	0.3	0.90	0.05	0.05
g_3 = "patient satisfaction"	0.2	0.68	0.20	0.12

Choose the (componentwise) weighted mean as goal aggregator:

$$(T^*, I^*, F^*) = \Big(\sum\nolimits_{g\in\mathcal{G}} w_g T_x(g),\ \sum\nolimits_{g\in\mathcal{G}} w_g I_x(g),\ \sum\nolimits_{g\in\mathcal{G}} w_g F_x(g)\Big).$$

A direct computation gives

$$T^* = 0.5\cdot 0.75 + 0.3\cdot 0.90 + 0.2\cdot 0.68 = 0.781,$$
$$I^* = 0.5\cdot 0.10 + 0.3\cdot 0.05 + 0.2\cdot 0.20 = 0.105,$$
$$F^* = 0.5\cdot 0.15 + 0.3\cdot 0.05 + 0.2\cdot 0.12 = 0.114.$$

With thresholds $\boldsymbol{\theta} = (\theta_T, \theta_I, \theta_F) = (0.70, 0.30, 0.20)$, we get $T^* \geq \theta_T$, $I^* \leq \theta_I$, and $F^* \leq \theta_F$; hence the ED is $\boldsymbol{\theta}$–neutrosophically successful at state x.

Example 10.3 (City Microgrid with Renewables)**.** Let $\mathcal{S}$ be a city microgrid (cf. [357, 358]) operating with solar/wind plus storage. Take $\mathcal{G} = \{g_1, g_2, g_3\}$ with weights $w_{g_1} = 0.4$, $w_{g_2} = 0.3$, $w_{g_3} = 0.3$:

goal g	w_g	$T_x(g)$	$I_x(g)$	$F_x(g)$
g_1 = "supply reliability"	0.4	0.80	0.10	0.10
g_2 = "cost efficiency"	0.3	0.60	0.15	0.25
g_3 = "CO_2 reduction"	0.3	0.85	0.05	0.10

Using the same weighted-mean aggregator,

$$T^* = 0.4 \cdot 0.80 + 0.3 \cdot 0.60 + 0.3 \cdot 0.85 = 0.755,$$

$$I^* = 0.4 \cdot 0.10 + 0.3 \cdot 0.15 + 0.3 \cdot 0.05 = 0.10,$$

$$F^* = 0.4 \cdot 0.10 + 0.3 \cdot 0.25 + 0.3 \cdot 0.10 = 0.145.$$

With thresholds $\boldsymbol{\theta} = (0.75, 0.20, 0.20)$ we have $T^* \geq 0.75$, $I^* \leq 0.20$, $F^* \leq 0.20$, therefore the microgrid is $\boldsymbol{\theta}$–neutrosophically successful at state x (despite cost uncertainty).

10.2 Neutrosophic truth tables

Neutrosophic truth tables assign triplets for each connective and proposition, capturing truth, indeterminacy, and falsity, enabling graded, paraconsistent evaluation semantics [352]

Definition 10.4 (Neutrosophic Truth Tables (Connectives by t–norm/t–conorm))**.** Fix a left–continuous t–norm $t : [0,1]^2 \to [0,1]$ and its dual t–conorm $s : [0,1]^2 \to [0,1]$. For neutrosophic values $v = (T, I, F)$ and $v' = (T', I', F')$, define the basic connectives componentwise:

$$\begin{aligned} v \wedge v' &:= \big(t(T,T'),\ s(I,I'),\ s(F,F')\big), \\ v \vee v' &:= \big(s(T,T'),\ t(I,I'),\ t(F,F')\big), \\ \neg v &:= \big(F,\ I,\ T\big) \quad \text{(involutive swap; optionally complement components by } x \mapsto 1-x). \end{aligned}$$

A *neutrosophic truth table* lists these outputs for the rows (v, v') of interest; the choice of (t, s) (e.g. product/$x + y - xy$, min/max) yields concrete operational tables usable in applications.

Remark 10.5 (Reading the Tables)**.** In $\wedge$, truth is conjunctive (via t), while both indeterminacy and falsity accumulate disjunctively (via s); in $\vee$, truth accumulates (via s) whereas indeterminacy and falsity meet conjunctively (via t). This pattern generalizes classical truth tables and supports refined n–valued variants.

Example 10.6 (Neutrosophic Truth-Table Row in Medical Testing (Two Tests for the Same Disease))**.** Consider the proposition D: "The patient has disease X." Two independent tests provide neutrosophic assessments:

$$v_A(D) = (T, I, F) = (0.80,\ 0.10,\ 0.20), \qquad v_B(D) = (0.60,\ 0.25,\ 0.30).$$

Choose t–norm $t = \min$ and t–conorm $s = \max$ (Gödel pair). The basic connectives, applied componentwise, yield

$$v_A(D) \wedge v_B(D) = \big(\min(0.80, 0.60),\ \max(0.10, 0.25),\ \max(0.20, 0.30)\big) = (0.60,\ 0.25,\ 0.30),$$

$$v_A(D) \vee v_B(D) = \big(\max(0.80, 0.60),\ \min(0.10, 0.25),\ \min(0.20, 0.30)\big) = (0.80,\ 0.10,\ 0.20),$$

$$\neg v_A(D) = (F, I, T) = (0.20,\ 0.10,\ 0.80).$$

Interpretation: a "both-tests-positive" policy ($\wedge$) retains the lower truth 0.60 and raises indeterminacy to 0.25; an "at-least-one-positive" policy ($\vee$) preserves the higher truth 0.80 and lowers indeterminacy to 0.10. The negation of the first test describes the evidential status of "no disease" for clinical differential diagnosis.

10.3 Neutrosophic Behavior Gradient

Behavior Gradient quantifies directional rates at which observable behavior improves, degrades, or becomes uncertain under changing stimuli, context, and structure (cf. [359]). Neutrosophic behavior gradient quantifies how a system's quasi-structure drives graded tendencies, across discrete, continuous, and hybrid dynamics under partial stimulation [353–355].

Definition 10.7 (Neutrosophic Behavior Gradient)**.** [353–355] Let $\mathcal{S}$ be a neutrosophic system with state space $X \subseteq \mathbb{R}^d$, input (stimulus) space U, and environment space E. A trajectory $t \mapsto x(t)$ evolves under a vector field $f : X \times U \times E \to \mathbb{R}^d$:

$$\dot{x}(t) = f\big(x(t), u(t), e(t)\big) \quad \text{(continuous time)}, \qquad x_{k+1} = F\big(x_k, u_k, e_k\big) \quad \text{(discrete time)}.$$

The *neutrosophic behavior fields*

$$\beta_T,\ \beta_I,\ \beta_F :\ X \times U \times E \longrightarrow [0,1]$$

assign, respectively, degrees of truth/achievement, indeterminacy/ambiguity, and falsity/failure to the instantaneous behavior of $\mathcal{S}$.

Continuous-time form. The *neutrosophic behavior gradient* is the triplet of directional rates

$$\mathcal{G}(x,u,e) \ :=\ \Big(\nabla\beta_T \cdot \tilde{f},\ \nabla\beta_I \cdot \tilde{f},\ \nabla\beta_F \cdot \tilde{f}\Big), \qquad \tilde{f} = (f, \dot{u}, \dot{e}),$$

i.e. for $\star \in \{T, I, F\}$, $\dfrac{d}{dt}\beta_\star(x(t),u(t),e(t)) = \nabla_x\beta_\star \cdot f + \nabla_u\beta_\star \cdot \dot{u} + \nabla_e\beta_\star \cdot \dot{e}$. If classical derivatives do not exist, replace by generalized (Dini/Clarke) derivatives, yielding a *quasi-gradient*.

Discrete-time form.

$$\mathcal{G}_k \ :=\ \Big(\Delta\beta_T,\ \Delta\beta_I,\ \Delta\beta_F\Big) \ :=\ \Big(\beta_T(x_{k+1},u_{k+1},e_{k+1}) - \beta_T(x_k,u_k,e_k),\ \text{idem for } I, F\Big).$$

Structural modulation. Let $\sigma : X \to [0,1]$ quantify the (quasi-)structural strength at x (how strongly the system's structure manifests). The *structure–modulated* gradient is

$$\mathcal{G}^{(\sigma)}(x,u,e) \ :=\ \sigma(x)\,\mathcal{G}(x,u,e).$$

This definition uniformly covers discrete, continuous, and hybrid systems (in the hybrid case one adds jump terms to $\mathcal{G}$ at switching times).

Example 10.8 (Personalized Tutoring with Adaptive Difficulty)**.** Consider a learning system with state $x = (m, c) \in [0,1] \times [0,\infty)$, where m is the learner's mastery and c the cognitive load. The input is $u = (d, h)$ with content difficulty $d \in [0,1]$ and hint rate $h \in [0,1]$. The environment $e = (n)$ models noise/fatigue $n \in [0,1]$.

Dynamics. Let $\dot{m} = a\,(1-m)\,s(d,m,h) - b\,n\,m$ and $\dot{c} = \alpha d - \beta h - \gamma c$, with $a, b, \alpha, \beta, \gamma > 0$ and

$$s(d,m,h) := \exp\!\big(-\eta(d-m)^2\big)\,\big(1+\lambda h\big), \quad \eta, \lambda > 0.$$

Neutrosophic behavior fields. Define an overload sigmoid $\sigma_c(z) := (1+e^{-\kappa z})^{-1}$ with $\kappa > 0$ and threshold $c_0 > 0$. Then set

$$\beta_T(x,u,e) \ =\ m\big(1-\sigma_c(c-c_0)\big), \qquad \beta_I(x,u,e) \ =\ 1-\exp\!\big(-\mu_1|d-m| - \mu_2 n\big),$$
$$\beta_F(x,u,e) \ =\ 1-\exp\!\big(-\lambda_1\sigma_c(c-c_0) - \lambda_2 n - \lambda_3|d-m|\big),$$

with $\mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_3 > 0$.

Neutrosophic behavior gradient (continuous time). Write $\sigma_c'(z) = \kappa\,\sigma_c(z)\big(1-\sigma_c(z)\big)$. The directional rates are

$$\frac{d}{dt}\beta_T = \underbrace{\big(1-\sigma_c(c-c_0)\big)}_{\partial\beta_T/\partial m}\dot{m} \ -\ \underbrace{m\,\sigma_c'(c-c_0)}_{\partial\beta_T/\partial c}\dot{c},$$

$$\frac{d}{dt}\beta_I = \underbrace{\mu_1\,\mathrm{sgn}(m-d)\,e^{-(\mu_1|d-m|+\mu_2 n)}}_{\partial\beta_I/\partial m}\dot{m}, \qquad \frac{d}{dt}\beta_F = \underbrace{\lambda_3\,\mathrm{sgn}(m-d)\,e^{-(\cdots)}}_{\partial\beta_F/\partial m}\dot{m} \ +\ \underbrace{\lambda_1\sigma_c'(c-c_0)\,e^{-(\cdots)}}_{\partial\beta_F/\partial c}\dot{c},$$

where $e^{-(\cdots)}$ abbreviates the corresponding exponential factor. Positive $\frac{d}{dt}\beta_T$ indicates behavior moving toward successful learning (more mastery, less overload); increases in β_I or β_F signal growing ambiguity or failure risk.

Example 10.9 (Autonomous Eco-Driving under Uncertain Traffic and Weather)**.** Let the vehicle state be $x = (v, h, s)$ with speed $v \geq 0$, headway $h > 0$, and battery state-of-charge $s \in [0, 1]$. The control is acceleration $u = (a)$, and the environment $e = (\rho, w, v_L, \sigma_{v_L})$ includes traffic density ρ, weather slipperiness $w \in [0, 1]$, lead-vehicle speed v_L, and its variability $\sigma_{v_L} \geq 0$.

Dynamics.

$$\dot{v} = a - \kappa v^2, \qquad \dot{h} = v_L - v, \qquad \dot{s} = -\varphi_1 v^2 - \varphi_2 a^2 - \varphi_3 w + \varphi_4 \max\{0, -a\}\, v,$$

with $\kappa, \varphi_i > 0$.

Neutrosophic behavior fields. Define safety and efficiency factors

$$\mathrm{safe}(v, h, w) = \exp\!\big(-\alpha \max\{0, \tfrac{v}{v_{\mathrm{safe}}(w)} - 1\}^2 - \beta \max\{0, h_{\mathrm{safe}} - h\}^2\big),$$

$$\mathrm{eff}(v, a, s) = \exp\!\big(-\gamma_1 v^2 - \gamma_2 a^2 - \gamma_3(1 - s)\big),$$

with $v_{\mathrm{safe}}(w)$ decreasing in w and $h_{\mathrm{safe}} > 0$. Then

$$\beta_T(x, u, e) = \mathrm{safe}(v, h, w) \cdot \mathrm{eff}(v, a, s), \quad \beta_I(x, u, e) = 1 - \exp\!\big(-\mu_1 \sigma_{v_L} - \mu_2 w - \mu_3 \rho\big),$$

$$\beta_F(x, u, e) = \sigma\Big(\eta\big(TTC_{\mathrm{th}} - \tfrac{h}{\max\{v - v_L, \varepsilon\}}\big)\Big), \quad \sigma(z) := (1 + e^{-z})^{-1},\ \varepsilon > 0.$$

Neutrosophic behavior gradient (continuous time). Using $\log \beta_T = \log \mathrm{safe} + \log \mathrm{eff}$,

$$\frac{d}{dt}\beta_T = \beta_T\Big(\frac{\partial \log \mathrm{safe}}{\partial v}\dot{v} + \frac{\partial \log \mathrm{safe}}{\partial h}\dot{h} + \frac{\partial \log \mathrm{eff}}{\partial v}\dot{v} + \frac{\partial \log \mathrm{eff}}{\partial a}\dot{a} + \frac{\partial \log \mathrm{eff}}{\partial s}\dot{s}\Big),$$

where, for instance,

$$\frac{\partial \log \mathrm{safe}}{\partial v} = -2\alpha \max\Big\{0, \frac{v}{v_{\mathrm{safe}}(w)} - 1\Big\} \cdot \frac{1}{v_{\mathrm{safe}}(w)}, \quad \frac{\partial \log \mathrm{safe}}{\partial h} = 2\beta \max\{0, h_{\mathrm{safe}} - h\} \cdot \mathbf{1}_{\{h < h_{\mathrm{safe}}\}},$$

$$\frac{\partial \log \mathrm{eff}}{\partial v} = -2\gamma_1 v, \quad \frac{\partial \log \mathrm{eff}}{\partial a} = -2\gamma_2 a, \quad \frac{\partial \log \mathrm{eff}}{\partial s} = -\gamma_3.$$

Similarly,

$$\frac{d}{dt}\beta_I = 0 \quad \text{(if } \sigma_{v_L}, w, \rho \text{ treated as exogenous constants over the step)},$$

and, writing $TTC = h/\max\{v - v_L, \varepsilon\}$,

$$\frac{d}{dt}\beta_F = \sigma\big(\eta(TTC_{\mathrm{th}} - TTC)\big)\big(1 - \sigma\big(\eta(TTC_{\mathrm{th}} - TTC)\big)\big)\,\eta\,\frac{d}{dt}\big(TTC_{\mathrm{th}} - TTC\big),$$

$$\frac{d}{dt}TTC = \frac{\dot{h}\max\{v - v_L, \varepsilon\} - h\,(\dot{v} - \dot{v}_L)\mathbf{1}_{\{v > v_L + \varepsilon\}}}{\max\{v - v_L, \varepsilon\}^2}.$$

A control update $a \mapsto a'$ that makes $\frac{d}{dt}\beta_T > 0$ while keeping $\frac{d}{dt}\beta_F \leq 0$ and β_I small pushes the behavior toward safer, more efficient driving under uncertainty.

10.4 Neutrosynthesis

Synthesis is the process of combining diverse elements into a coherent whole, reconciling tensions and producing emergent structures, insights, functions [360–364]. Neutrosynthesis integrates analysis and synthesis by jointly processing truth, indeterminacy, and falsity, reconciling conflicts to produce context-sensitive, balanced conclusions outcomes [60,365,366].

Definition 10.10 (Neutrosynthesis)**.** [365] Consider a family of information items (data/rules/agents) indexed by J, each endowed with a neutrosophic triplet $v_j = (T_j, I_j, F_j) \in [0, 1]^3$. Partition $J = J_{\mathrm{pr}} \sqcup J_{\mathrm{an}} \sqcup J_{\mathrm{np}}$ into *principles* (promote truth), *anti-principles* (promote falsity), and *nonprinciples* (promote indeterminacy). Let nonnegative weights w_j satisfy $\sum_{j \in J_{\mathrm{pr}}} w_j = \sum_{j \in J_{\mathrm{an}}} w_j = \sum_{j \in J_{\mathrm{np}}} w_j = 1$ (if a block is empty, omit its term). The *neutrosynthesis operator* NSyn produces an aggregate triplet $\mathsf{NSyn}\big((v_j)_{j \in J}\big) = (T^*, I^*, F^*) \in [0, 1]^3$ by

$$T^* = \min\Big\{1,\ \Big(\sum_{j \in J_{\mathrm{pr}}} w_j T_j\Big) \cdot \Big(1 - \sum_{j \in J_{\mathrm{an}}} w_j F_j\Big)\Big\},$$

$$F^* = \min\Big\{1,\ \Big(\sum_{j \in J_{\mathrm{an}}} w_j F_j\Big) \cdot \Big(1 - \sum_{j \in J_{\mathrm{pr}}} w_j T_j\Big)\Big\},$$

$$I^* = \min\Big\{1,\ \sum_{j \in J_{\mathrm{np}}} w_j I_j \ +\ \gamma\Big(\sum_{j \in J_{\mathrm{pr}}} w_j T_j\Big)\Big(\sum_{j \in J_{\mathrm{an}}} w_j F_j\Big)\Big\},$$

with a design parameter $\gamma \in [0, 1]$ capturing indeterminacy generated by confrontation between principles and anti-principles. The operator is monotone increasing in each T_j, and monotone decreasing (resp. increasing) in antagonistic F_j for T^* (resp. F^*), while I^* accumulates both intrinsic unknowns and conflict–induced ambiguity.

Example 10.11 (Medical Triage: Aggregating Conflicting Clinical Evidence via Neutrosynthesis)**.** We assess the hypothesis *"acute bacterial pneumonia"* from multiple sources. Partition the information items $J = J_{\text{pr}} \sqcup J_{\text{an}} \sqcup J_{\text{np}}$ as:

$$\begin{aligned} J_{\text{pr}} &= \{\text{X-ray consolidation } (j_1),\ \text{elevated CRP } (j_2)\}, \\ J_{\text{an}} &= \{\text{influenza A positive } (j_3)\}, \\ J_{\text{np}} &= \{\text{low radiograph quality } (j_4),\ \text{atypical presentation } (j_5)\}. \end{aligned}$$

Assign neutrosophic triplets $v_j = (T_j, I_j, F_j) \in [0,1]^3$ and within-block weights $\sum_{j \in J_{\text{pr}}} w_j = \sum_{j \in J_{\text{an}}} w_j = \sum_{j \in J_{\text{np}}} w_j = 1$:

j	v_j	w_j
j_1	$(T_1, I_1, F_1) = (0.80, 0.10, 0.10)$	0.6
j_2	$(0.70, 0.20, 0.10)$	0.4
j_3	$(0.10, 0.30, 0.60)$	1.0
j_4	$(0.20, 0.50, 0.30)$	0.7
j_5	$(0.10, 0.30, 0.30)$	0.3

Using the neutrosynthesis operator with $\gamma = 0.5$,

$$\begin{aligned} \textstyle\sum_{j \in J_{\text{pr}}} w_j T_j &= 0.6 \cdot 0.80 + 0.4 \cdot 0.70 = 0.76, \\ \textstyle\sum_{j \in J_{\text{an}}} w_j F_j &= 1.0 \cdot 0.60 = 0.60, \\ \textstyle\sum_{j \in J_{\text{np}}} w_j I_j &= 0.7 \cdot 0.50 + 0.3 \cdot 0.30 = 0.44, \end{aligned}$$

$$\begin{aligned} T^* &= \min\{1,\ 0.76 \cdot (1 - 0.60)\} = 0.304, \\ F^* &= \min\{1,\ 0.60 \cdot (1 - 0.76)\} = 0.144, \\ I^* &= \min\{1,\ 0.44 + 0.5 \cdot (0.76 \cdot 0.60)\} = 0.668. \end{aligned}$$

Interpretation. Positive imaging and inflammatory markers are partially offset by a strong viral alternative (F^*), while degraded image quality and atypical features raise ambiguity (I^*). The aggregate suggests "possible pneumonia, high indeterminacy; consider antiviral/await clarifying tests."

Example 10.12 (Cybersecurity Incident Triage under Uncertain Context)**.** We evaluate the hypothesis *"ongoing intrusion"* from heterogeneous detectors and context. Decompose J as:

$$\begin{aligned} J_{\text{pr}} &= \{\text{signature IDS alert } (j_1),\ \text{UEBA anomaly } (j_2)\}, \\ J_{\text{an}} &= \{\text{approved maintenance window } (j_3),\ \text{benign reputation signal } (j_4)\}, \\ J_{\text{np}} &= \{\text{encrypted traffic masking } (j_5),\ \text{clock drift/telemetry gaps } (j_6)\}. \end{aligned}$$

Triplets and normalized within-block weights:

j	$v_j = (T_j, I_j, F_j)$	w_j
j_1	$(0.90, 0.05, 0.10)$	0.5
j_2	$(0.70, 0.20, 0.10)$	0.5
j_3	$(0.10, 0.20, 0.50)$	0.6
j_4	$(0.10, 0.20, 0.40)$	0.4
j_5	$(0.10, 0.60, 0.10)$	0.7
j_6	$(0.10, 0.20, 0.10)$	0.3

With $\gamma = 0.4$,

$$\begin{aligned} \textstyle\sum_{j \in J_{\text{pr}}} w_j T_j &= 0.5 \cdot 0.90 + 0.5 \cdot 0.70 = 0.80, \\ \textstyle\sum_{j \in J_{\text{an}}} w_j F_j &= 0.6 \cdot 0.50 + 0.4 \cdot 0.40 = 0.46, \\ \textstyle\sum_{j \in J_{\text{np}}} w_j I_j &= 0.7 \cdot 0.60 + 0.3 \cdot 0.20 = 0.48, \end{aligned}$$

$$\begin{aligned} T^* &= \min\{1,\ 0.80 \cdot (1 - 0.46)\} = 0.432, \\ F^* &= \min\{1,\ 0.46 \cdot (1 - 0.80)\} = 0.092, \\ I^* &= \min\{1,\ 0.48 + 0.4 \cdot (0.80 \cdot 0.46)\} = 0.6272. \end{aligned}$$

Strong attack indicators are tempered by plausible benign explanations (maintenance, reputation), while encryption and telemetry gaps drive residual uncertainty. Recommended action: raise priority for containment with additional evidence collection to resolve I^*.

10.5 Neutrosophic Synergy

Synergy is the combined effect of interacting agents exceeding their individual contributions, via cooperation, coordination, and complementary capabilities under shared goals [367–372]. Neutrosophic synergy models partially combined efforts: agents cooperate, oppose, or remain neutral in degrees, yielding aggregated effects via neutrosophic operations [353, 354]. As related work, several papers have investigated the relationship between fuzzy logic and synergy [373–377].

Definition 10.13 (Neutrosophic Synergy)**.** [353, 354] Let $K \geq 2$ participants have capability scores $c_i \in [0,1]$ ($i = 1, \ldots, K$). For each pair (i, j) with $i < j$, let interaction degrees

$$\rho_{ij}^{+},\ \rho_{ij}^{0},\ \rho_{ij}^{-} \in [0,1], \qquad \rho_{ij}^{+} + \rho_{ij}^{0} + \rho_{ij}^{-} \leq 1,$$

measure, respectively, cooperation, neutrality, and antagonism in their joint work; let nonnegative pair weights w_{ij} be given. The *neutrosophic synergy* of the team is the triplet

$$\begin{aligned} T_{\text{syn}} &= \frac{1}{W} \sum_{1 \leq i < j \leq K} w_{ij}\, c_i c_j\, \rho_{ij}^{+}, \\ I_{\text{syn}} &= \frac{1}{W} \sum_{1 \leq i < j \leq K} w_{ij}\, c_i c_j\, \rho_{ij}^{0}, \qquad W := \sum_{1 \leq i < j \leq K} w_{ij}\, c_i c_j\ (>0). \\ F_{\text{syn}} &= \frac{1}{W} \sum_{1 \leq i < j \leq K} w_{ij}\, c_i c_j\, \rho_{ij}^{-}, \end{aligned}$$

Thus $T_{\text{syn}}, I_{\text{syn}}, F_{\text{syn}} \in [0,1]$, they sum to at most 1, and they quantify, respectively, the effective degree of cooperative gain, neutral coexistence, and antagonistic loss among partially joined forces. The construction is symmetric in the participants and monotone in both capabilities and the corresponding interaction degrees.

Example 10.14 (Hospital Emergency Response: Cross-Agency Team Synergy)**.** Three participants collaborate during a respiratory outbreak: EMS ($i = 1$, capability $c_1 = 0.90$), Hospital ER ($i = 2$, $c_2 = 0.85$), and Public Health ($i = 3$, $c_3 = 0.70$). Take pair weights $w_{12} = w_{13} = w_{23} = 1$ and interaction degrees

$$\begin{aligned} (\rho_{12}^{+}, \rho_{12}^{0}, \rho_{12}^{-}) &= (0.80, 0.10, 0.05), \\ (\rho_{13}^{+}, \rho_{13}^{0}, \rho_{13}^{-}) &= (0.50, 0.30, 0.10), \qquad \text{(each triple sums } \leq 1). \\ (\rho_{23}^{+}, \rho_{23}^{0}, \rho_{23}^{-}) &= (0.60, 0.20, 0.15), \end{aligned}$$

The normalization factor is

$$W = \sum_{1 \leq i < j \leq 3} w_{ij}\, c_i c_j = (0.90 \cdot 0.85) + (0.90 \cdot 0.70) + (0.85 \cdot 0.70) = 0.765 + 0.630 + 0.595 = 1.990.$$

Cooperative, neutral, and antagonistic numerators are

$$\begin{aligned} N_T &= \sum_{i<j} w_{ij}\, c_i c_j\, \rho_{ij}^{+} = 0.765 \cdot 0.80 + 0.630 \cdot 0.50 + 0.595 \cdot 0.60 \\ &= 0.612 + 0.315 + 0.357 = 1.284, \\ N_I &= \sum_{i<j} w_{ij}\, c_i c_j\, \rho_{ij}^{0} = 0.765 \cdot 0.10 + 0.630 \cdot 0.30 + 0.595 \cdot 0.20 \\ &= 0.0765 + 0.189 + 0.119 = 0.3845, \\ N_F &= \sum_{i<j} w_{ij}\, c_i c_j\, \rho_{ij}^{-} = 0.765 \cdot 0.05 + 0.630 \cdot 0.10 + 0.595 \cdot 0.15 \\ &= 0.03825 + 0.063 + 0.08925 = 0.1905. \end{aligned}$$

Hence the neutrosophic synergy triplet is

$$(T_{\text{syn}}, I_{\text{syn}}, F_{\text{syn}}) = \left(\tfrac{N_T}{W}, \tfrac{N_I}{W}, \tfrac{N_F}{W}\right) \approx (0.646,\ 0.193,\ 0.096),$$

which sums to $\approx 0.935 \leq 1$. *Interpretation:* strong cooperative gain between EMS–ER dominates; data quality limits and jurisdictional frictions introduce moderate indeterminacy and a small antagonistic component.

Example 10.15 (Cross-Company AI Product Launch: R&D–Product–Legal–Security)**.** Four stakeholders build a regulated AI feature: Data Science ($1, c_1 = 0.85$), Product ($2, c_2 = 0.80$), Legal/Compliance ($3, c_3 = 0.70$), IT Security ($4, c_4 = 0.75$). Pair weights emphasize critical couplings:

$$(w_{12}, w_{13}, w_{14}, w_{23}, w_{24}, w_{34}) = (1.2,\ 0.8,\ 1.0,\ 1.0,\ 0.9,\ 1.1).$$

Interaction degrees (cooperation, neutrality, antagonism) for each pair are

$$(\rho^+_{12}, \rho^0_{12}, \rho^-_{12}) = (0.75, 0.15, 0.05), \qquad (\rho^+_{13}, \rho^0_{13}, \rho^-_{13}) = (0.40, 0.40, 0.15),$$
$$(\rho^+_{14}, \rho^0_{14}, \rho^-_{14}) = (0.50, 0.30, 0.15), \qquad (\rho^+_{23}, \rho^0_{23}, \rho^-_{23}) = (0.45, 0.35, 0.15),$$
$$(\rho^+_{24}, \rho^0_{24}, \rho^-_{24}) = (0.55, 0.25, 0.15), \qquad (\rho^+_{34}, \rho^0_{34}, \rho^-_{34}) = (0.60, 0.25, 0.10).$$

Compute

$$\begin{aligned} W = \sum_{i<j} w_{ij}\, c_i c_j &= 1.2(0.85 \cdot 0.80) + 0.8(0.85 \cdot 0.70) + 1.0(0.85 \cdot 0.75) \\ &\quad + 1.0(0.80 \cdot 0.70) + 0.9(0.80 \cdot 0.75) + 1.1(0.70 \cdot 0.75) \\ &= 0.816 + 0.476 + 0.6375 + 0.56 + 0.54 + 0.5775 = 3.607. \end{aligned}$$

The numerators are

$$\begin{aligned} N_T &= 0.816 \cdot 0.75 + 0.476 \cdot 0.40 + 0.6375 \cdot 0.50 + 0.56 \cdot 0.45 + 0.54 \cdot 0.55 + 0.5775 \cdot 0.60 \\ &= 0.612 + 0.1904 + 0.31875 + 0.252 + 0.297 + 0.3465 = 2.01665, \\ N_I &= 0.816 \cdot 0.15 + 0.476 \cdot 0.40 + 0.6375 \cdot 0.30 + 0.56 \cdot 0.35 + 0.54 \cdot 0.25 + 0.5775 \cdot 0.25 \\ &= 0.1224 + 0.1904 + 0.19125 + 0.196 + 0.135 + 0.144375 = 0.979425, \\ N_F &= 0.816 \cdot 0.05 + 0.476 \cdot 0.15 + 0.6375 \cdot 0.15 + 0.56 \cdot 0.15 + 0.54 \cdot 0.15 + 0.5775 \cdot 0.10 \\ &= 0.0408 + 0.0714 + 0.095625 + 0.084 + 0.081 + 0.05775 = 0.430575. \end{aligned}$$

Therefore

$$(T_{\text{syn}}, I_{\text{syn}}, F_{\text{syn}}) = \left(\tfrac{2.01665}{3.607}, \tfrac{0.979425}{3.607}, \tfrac{0.430575}{3.607} \right) \approx (0.559,\ 0.272,\ 0.119),$$

with total $\approx 0.950 \le 1$. *Interpretation:* substantial cooperative gain emerges from Data Science–Product and Legal–Security pairs; compliance reviews and security hardening inject delay/ambiguity (neutrality) and some antagonism due to scope constraints.

10.6 Neutrosophic Interaction

Neutrosophic interactions encode supportive, neutral, and antagonistic influences between entities using triplets, composing along paths with t-norm and t-conorm semantics [353].

Definition 10.16 (Neutrosophic Triplets and Basic Operations)**.** [295–297, 378, 379] Let $[0, 1]$ denote the closed unit interval and fix a (left–continuous) t–norm $t : [0, 1]^2 \to [0, 1]$ with dual t–conorm $s : [0, 1]^2 \to [0, 1]$. A *neutrosophic truth value* is a triplet $v = (T, I, F) \in [0, 1]^3$, interpreted as degrees of truth/achievement (T), indeterminacy/ambiguity (I), and falsity/failure (F). When needed, we assume the normalized constraint $T + I + F \le 1$. We write $v \oplus v' := (\, s(T, T'),\ s(I, I'),\ s(F, F')\,)$ and $v \otimes v' := (\, t(T, T'),\ t(I, I'),\ t(F, F')\,)$ for componentwise aggregation.

Example 10.17 (Basic Triplets and Operations with Gödel Pair $(t, s) = (\min, \max)$)**.** Let $t = \min$ and $s = \max$. Consider two neutrosophic truth values

$$v = (T, I, F) = (0.70,\ 0.20,\ 0.10), \qquad v' = (0.55,\ 0.30,\ 0.25).$$

Then the componentwise t–norm / t–conorm aggregations are

$$v \otimes v' = (\min\{0.70, 0.55\},\ \min\{0.20, 0.30\},\ \min\{0.10, 0.25\}) = (0.55,\ 0.20,\ 0.10),$$

$$v \oplus v' = (\max\{0.70, 0.55\},\ \max\{0.20, 0.30\},\ \max\{0.10, 0.25\}) = (0.70,\ 0.30,\ 0.25).$$

Interpretation: conjunctive evidence tightens to the lower agreeing degrees, while disjunctive evidence retains the larger support/uncertainty/failure components.

Definition 10.18 (Pairwise Neutrosophic Interaction Kernel)**.** Let U be a nonempty set of entities (agents, variables, or states). A *neutrosophic interaction kernel* is a map

$$\Lambda : U \times U \longrightarrow [0, 1]^3, \qquad (i, j) \longmapsto \Lambda(i, j) = (\lambda_T(i, j), \lambda_I(i, j), \lambda_F(i, j)),$$

where λ_T measures cooperative/supportive influence of i on j, λ_F measures antagonistic/oppositional influence, and λ_I measures neutral/uncertain influence. We allow directionality (i.e. generally $\Lambda(i, j) \neq \Lambda(j, i)$) and either the local normalization $\lambda_T + \lambda_I + \lambda_F \le 1$ or no normalization (depending on the application).

Example 10.19 (A Directed Neutrosophic Interaction Kernel in Healthcare)**.** Let $U = \{\mathrm{D}, \mathrm{P}, \mathrm{A}\}$ denote Doctor, Patient, and AI Algorithm. Define the kernel $\Lambda : U \times U \to [0, 1]^3$ by the following (ordered as $(\lambda_T, \lambda_I, \lambda_F)$):

$$\Lambda(\mathrm{D}, \mathrm{P}) = (0.85, 0.10, 0.05), \quad \Lambda(\mathrm{A}, \mathrm{P}) = (0.60, 0.25, 0.10), \quad \Lambda(\mathrm{D}, \mathrm{A}) = (0.40, 0.45, 0.10),$$

$$\Lambda(\mathrm{P}, \mathrm{D}) = (0.30, 0.50, 0.10), \quad \Lambda(\mathrm{A}, \mathrm{D}) = (0.20, 0.50, 0.20), \quad \Lambda(\mathrm{P}, \mathrm{A}) = (0.35, 0.40, 0.15),$$

and set any unspecified pair to $(0, 0, 0)$ for simplicity. Here doctor→patient is strongly supportive; algorithm→patient is moderately supportive with some uncertainty; doctor→algorithm is largely neutral (model calibration). Directionality captures asymmetric influence (e.g., patient feedback to doctor is weaker and more uncertain).

Definition 10.20 (Neutrosophic Interaction Update (Message Passing))**.** Let each $u \in U$ carry a state $v(u) = (T(u), I(u), F(u)) \in [0,1]^3$. For a fixed $j \in U$ and a (possibly weighted) neighborhood $N(j) \subseteq U \setminus \{j\}$ with nonnegative weights w_{ij}, define the *incoming message* from i to j by

$$m_{i\to j} := \big(t(T(i), \lambda_T(i,j)),\ t(T(i), \lambda_I(i,j)),\ t(T(i), \lambda_F(i,j))\big).$$

The one–step neutrosophic update at j is

$$\begin{aligned} T^+(j) &= s\Big(T(j),\ \textstyle\bigoplus_{i\in N(j)} w_{ij}\, m^{(T)}_{i\to j}\Big),\\ F^+(j) &= s\Big(F(j),\ \textstyle\bigoplus_{i\in N(j)} w_{ij}\, m^{(F)}_{i\to j}\Big),\\ I^+(j) &= s\Big(I(j),\ \textstyle\bigoplus_{i\in N(j)} w_{ij}\, m^{(I)}_{i\to j},\ \gamma\cdot \bigoplus_{i\in N(j)} w_{ij}\, t\big(m^{(T)}_{i\to j}, m^{(F)}_{i\to j}\big)\Big), \end{aligned}$$

where $m^{(T)}_{i\to j}, m^{(I)}_{i\to j}, m^{(F)}_{i\to j}$ denote the T-, I-, F-components of $m_{i\to j}$, $\oplus$ is componentwise s-aggregation across $i \in N(j)$, and $\gamma \in [0,1]$ controls conflict-induced indeterminacy.

Example 10.21 (One-Step Neutrosophic Message Passing Update on a Patient Node)**.** Use $t = \min$, $s = \max$, and $\gamma = 0.5$. Let $j = \mathrm{P}$ (patient) have current state

$$v(j) = (T(j), I(j), F(j)) = (0.50,\ 0.30,\ 0.20).$$

Neighbors are $N(j) = \{\mathrm{D}, \mathrm{A}\}$ with weights $w_{\mathrm{D}j} = 0.6$, $w_{\mathrm{A}j} = 0.4$. Suppose the sending nodes have truth components $T(\mathrm{D}) = 0.90$, $T(\mathrm{A}) = 0.80$, and use the kernel from the previous example:

$$\Lambda(\mathrm{D},\mathrm{P}) = (0.85, 0.10, 0.05), \qquad \Lambda(\mathrm{A},\mathrm{P}) = (0.60, 0.25, 0.10).$$

Messages are

$$\begin{aligned} m_{\mathrm{D}\to j} &= \big(\min(0.90,0.85),\ \min(0.90,0.10),\ \min(0.90,0.05)\big) = (0.85, 0.10, 0.05),\\ m_{\mathrm{A}\to j} &= \big(\min(0.80,0.60),\ \min(0.80,0.25),\ \min(0.80,0.10)\big) = (0.60, 0.25, 0.10). \end{aligned}$$

Weighted incoming components:

$$\max\{0.6{\cdot}0.85,\ 0.4{\cdot}0.60\} = \max\{0.51,\ 0.24\} = 0.51, \quad \max\{0.6{\cdot}0.10,\ 0.4{\cdot}0.25\} = 0.10,$$

$$\max\{0.6{\cdot}0.05,\ 0.4{\cdot}0.10\} = 0.04.$$

Conflict terms per neighbor $\min(m^{(T)}, m^{(F)})$ give $\{0.05,\ 0.10\}$, hence

$$\max\{0.6{\cdot}0.05,\ 0.4{\cdot}0.10\} = 0.04, \qquad \gamma\cdot(\cdot) = 0.5\cdot 0.04 = 0.02.$$

Therefore the updated state at j is

$$T^+(j) = \max\{0.50,\ 0.51\} = 0.51, \quad F^+(j) = \max\{0.20,\ 0.04\} = 0.20, \quad I^+(j) = \max\{0.30,\ 0.10,\ 0.02\} = 0.30.$$

Interpretation: strong doctor support slightly raises the patient's T; uncertainty and failure do not increase this round.

Definition 10.22 (Neutrosophic Interaction Matrix and Composition)**.** Index $U = \{1, \dots, n\}$ and collect Λ into three matrices $L_T = (\lambda_T(i,j))_{i,j}$, $L_I = (\lambda_I(i,j))_{i,j}$, $L_F = (\lambda_F(i,j))_{i,j}$. Given two interaction kernels A and B on U with components A_T, A_I, A_F and B_T, B_I, B_F, their *(relational) neutrosophic composition* $C = B \circ A$ is defined entrywise by

$$\begin{aligned} C_T(i,j) &= \sup_k\ t\big(A_T(i,k),\ B_T(k,j)\big),\\ C_F(i,j) &= \sup_k\ t\big(A_F(i,k),\ B_F(k,j)\big),\\ C_I(i,j) &= \sup_k\ s\Big(t\big(A_I(i,k), B_I(k,j)\big),\ t\big(A_T(i,k), B_F(k,j)\big),\ t\big(A_F(i,k), B_T(k,j)\big)\Big). \end{aligned}$$

Thus supportive paths compose through t and sup (max–product type), oppositional paths compose analogously, while indeterminacy accumulates from indeterminacy–to–indeterminacy and from cross–type conflicts (support followed by opposition, or vice versa).

Example 10.23 (Composing Two Interaction Kernels on a Two-Node System)**.** Let $U = \{1, 2\}$ and choose $t = \min$, $s = \max$, sup = max. Define kernels A (stage 1) and B (stage 2):

$$A_T = \begin{pmatrix} 0.2 & 0.6 \\ 0.0 & 0.3 \end{pmatrix},\ A_I = \begin{pmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{pmatrix},\ A_F = \begin{pmatrix} 0.0 & 0.2 \\ 0.4 & 0.0 \end{pmatrix};\quad B_T = \begin{pmatrix} 0.1 & 0.4 \\ 0.5 & 0.5 \end{pmatrix},\ B_I = \begin{pmatrix} 0.2 & 0.1 \\ 0.4 & 0.3 \end{pmatrix},\ B_F = \begin{pmatrix} 0.0 & 0.3 \\ 0.2 & 0.1 \end{pmatrix}.$$

For the composition $C = B \circ A$, compute the $(1,2)$–entry.

$$C_T(1,2) = \max_{k\in\{1,2\}} \min\{A_T(1,k), B_T(k,2)\} = \max\{\min(0.2,0.4),\ \min(0.6,0.5)\} = \max\{0.2, 0.5\} = 0.5.$$

$$C_F(1,2) = \max_k \min\{A_F(1,k), B_F(k,2)\} = \max\{\min(0.0, 0.3),\ \min(0.2, 0.1)\} = \max\{0, 0.1\} = 0.1.$$

For indeterminacy,

$$\begin{aligned} C_I(1,2) &= \max_k \ \max\Big\{ \min(A_I(1,k), B_I(k,2)),\ \min(A_T(1,k), B_F(k,2)),\ \min(A_F(1,k), B_T(k,2)) \Big\} \\ &= \max\Big\{ \max\{\max(0.1, 0.3),\ \max(0.2, 0.1),\ \max(0.0, 0.2)\},\ \max\{\max(0.3, 0.1),\ \max(0.1, 0.2),\ \max(0.2, 0.5)\} \Big\} \\ &= \max\Big\{ \max\{0.3, 0.2, 0.2\},\ \max\{0.3, 0.2, 0.5\} \Big\} = \max\{0.3, 0.5\} = 0.5, \end{aligned}$$

where the first inner block corresponds to $k = 1$ and the second to $k = 2$. Thus $(C_T, C_I, C_F)(1,2) = (0.5,\ 0.5,\ 0.1)$: there is a strong supportive path $1 \to 2$ via $k = 2$, while cross-type interactions also raise indeterminacy; opposition remains limited. (Other entries are computed analogously.)

10.7 Neutrosophic Body–Mind–Soul–Spirit Fluidity

Neutrosophic Body–Mind–Soul–Spirit Fluidity models four coupled states, each with truth, indeterminacy, and falsity levels, evolving interactively to describe holistic psycho-spiritual balance.

Definition 10.24 (Neutrosophic Body–Mind–Soul–Spirit Fluidity (NBMSSF))**.** Let $\mathcal{X} = \{\text{Body}, \text{Mind}, \text{Soul}, \text{Spirit}\}$. NBMSSF is a mathematical scheme with four coupled components $X \in \mathcal{X}$. Each component carries a neutrosophic triad (T_X, I_X, F_X) and interacts with the other three components as follows.

(1) Neutrosophic triad (local state). For every $X \in \mathcal{X}$,

$$\begin{aligned} T_X &\in [0,1] \quad \text{(degree of truth)}, \\ I_X &\in [0,1] \quad \text{(degree of indeterminacy)}, \\ F_X &\in [0,1] \quad \text{(degree of falsity)}, \\ T_X + I_X + F_X &= 1. \end{aligned}$$

(2) Coupling / fluidity map. Writing $\{X, Y, Z, W\} = \mathcal{X}$, the instantaneous "fluidity" of X is encoded by an application–dependent influence function

$$\mathcal{F}_X \ = \ f_X\big(T_Y, I_Y, F_Y,\ T_Z, I_Z, F_Z,\ T_W, I_W, F_W\big),$$

which aggregates the neutrosophic states of the *other* three components.

(3) Interdependency dynamics. The evolution of each triad is governed by a system of differential equations

$$\begin{aligned} \frac{dT_X}{dt} &= g_{T,X}(T, I, F), \\ \frac{dI_X}{dt} &= g_{I,X}(T, I, F), \\ \frac{dF_X}{dt} &= g_{F,X}(T, I, F), \end{aligned}$$

where $g_{T,X}, g_{I,X}, g_{F,X}$ may include cross–component interactions. For instance, one may have

$$g_{T,X} = \alpha_X T_Y - \beta_X F_W,$$

with sensitivity coefficients α_X, β_X determined by context.

(4) Global state matrix. At time t, the overall configuration is the 4×3 matrix

$$\mathbf{S}(t) = \begin{bmatrix} T_{\text{Body}} & I_{\text{Body}} & F_{\text{Body}} \\ T_{\text{Mind}} & I_{\text{Mind}} & F_{\text{Mind}} \\ T_{\text{Soul}} & I_{\text{Soul}} & F_{\text{Soul}} \\ T_{\text{Spirit}} & I_{\text{Spirit}} & F_{\text{Spirit}} \end{bmatrix}.$$

(5) Component score / characteristic functional. Each component admits a linear assessment

$$\Phi_X(T_X, I_X, F_X) = \alpha_X T_X + \beta_X I_X + \gamma_X F_X,$$

where the weights $\alpha_X, \beta_X, \gamma_X$ are fixed by the application.

(6) Global balance constraint. Consistency across the four components is expressed by

$$\sum_{X \in \mathcal{X}} (T_X + I_X + F_X) = 4,$$

which is compatible with the local normalization $T_X + I_X + F_X = 1$ for each X.

Example 10.25 (NBMSSF in a Graduate Exam Week)**.** A graduate student manages well-being across four facets—Body, Mind, Soul (values/purpose), and Spirit (community/support)—during a demanding exam week. We model the four facets by neutrosophic triads and one short intervention window of length $\Delta t = \frac{1}{2}$ (half a day).

Initial state (before interventions).

$$\mathbf{S}(0) = \begin{bmatrix} T_{\text{Body}} & I_{\text{Body}} & F_{\text{Body}} \\ T_{\text{Mind}} & I_{\text{Mind}} & F_{\text{Mind}} \\ T_{\text{Soul}} & I_{\text{Soul}} & F_{\text{Soul}} \\ T_{\text{Spirit}} & I_{\text{Spirit}} & F_{\text{Spirit}} \end{bmatrix} = \begin{bmatrix} 0.40 & 0.30 & 0.30 \\ 0.50 & 0.30 & 0.20 \\ 0.60 & 0.20 & 0.20 \\ 0.70 & 0.20 & 0.10 \end{bmatrix}.$$

Interpretation: the student feels physically tired (Body $T = 0.40$), cognitively engaged (Mind $T = 0.50$), purposeful (Soul $T = 0.60$), and well-supported (Spirit $T = 0.70$).

Coupling/dynamics (one step). For each $X \in \{\text{Body}, \text{Mind}, \text{Soul}, \text{Spirit}\}$ let $\{X, Y, Z, W\}$ be the four components. Define the cross-influences

$$\overline{T}^{\backslash X} = \frac{T_Y + T_Z + T_W}{3}, \qquad \overline{F}^{\backslash X} = \frac{F_Y + F_Z + F_W}{3}, \qquad \overline{I}^{\backslash X} = \frac{I_Y + I_Z + I_W}{3}.$$

With parameters $\alpha = 0.3,\ \beta = 0.2,\ \gamma = 0.5$, use

$$\frac{dT_X}{dt} = \alpha\, \overline{T}^{\backslash X} - \beta\, \overline{F}^{\backslash X}, \qquad \frac{dF_X}{dt} = -\frac{dT_X}{dt}, \qquad \frac{dI_X}{dt} = -\gamma\big(I_X - \overline{I}^{\backslash X}\big),$$

followed by a simplex–projection (renormalization)

$$\text{norm}(a, b, c) := \frac{(a, b, c)}{a + b + c}$$

to enforce $T_X + I_X + F_X = 1$ after the step.

Illustration (Body row, computed explicitly). For Body, from $\mathbf{S}(0)$:

$$\overline{T}^{\backslash \text{Body}} = \frac{0.50 + 0.60 + 0.70}{3} = 0.6, \quad \overline{F}^{\backslash \text{Body}} = \frac{0.20 + 0.20 + 0.10}{3} = \frac{1}{6}, \quad \overline{I}^{\backslash \text{Body}} = \frac{0.30 + 0.20 + 0.20}{3} = \frac{7}{30}.$$

Hence

$$\frac{dT_{\text{Body}}}{dt} = 0.3 \cdot 0.6 - 0.2 \cdot \frac{1}{6} = \frac{11}{75}, \quad \frac{dF_{\text{Body}}}{dt} = -\frac{11}{75}, \quad \frac{dI_{\text{Body}}}{dt} = -0.5\Big(0.30 - \frac{7}{30}\Big) = -\frac{1}{30}.$$

A half-day update ($\Delta t = \frac{1}{2}$) gives the unnormalized values

$$T'_{\text{Body}} = 0.40 + \tfrac{1}{2} \cdot \frac{11}{75} = \frac{71}{150}, \quad F'_{\text{Body}} = 0.30 - \tfrac{1}{2} \cdot \frac{11}{75} = \frac{17}{75}, \quad I'_{\text{Body}} = 0.30 - \tfrac{1}{2} \cdot \frac{1}{30} = \frac{17}{60},$$

whose sum is $\frac{59}{60}$. After renormalization,

$$(T_{\text{Body}}, I_{\text{Body}}, F_{\text{Body}})\Big|_{t=\Delta t} = \text{norm}\Big(\frac{71}{150}, \frac{17}{60}, \frac{17}{75}\Big) = \Big(\frac{142}{295}, \frac{17}{59}, \frac{68}{295}\Big).$$

Resulting state after one half-day. Applying the same step to all four rows yields

$$\mathbf{S}\Big(\tfrac{1}{2}\Big) = \begin{bmatrix} \frac{142}{295} & \frac{17}{59} & \frac{68}{295} \\ \frac{339}{590} & \frac{17}{59} & \frac{81}{590} \\ \frac{198}{305} & \frac{13}{61} & \frac{42}{305} \\ \frac{451}{610} & \frac{13}{61} & \frac{29}{610} \end{bmatrix} \approx \begin{bmatrix} 0.481 & 0.288 & 0.231 \\ 0.575 & 0.288 & 0.137 \\ 0.649 & 0.213 & 0.138 \\ 0.739 & 0.213 & 0.048 \end{bmatrix}.$$

A short intervention mix (hydration and a light jog for Body; focused 25-min review for Mind; 10-min values journaling for Soul; supportive call with a mentor for Spirit) produces: (i) higher T and lower F for Body and Spirit, (ii) steadier Mind with reduced F, and (iii) a modest rise in Soul's T with controlled I. The renormalization keeps each triad on the unit simplex.

Chapter 11: Conclusion

In this paper, we reviewed and examined Neutrosociology, Biology, Chemistry, Physics, and Psychology, along with extended concepts formulated through the Neutrosophic Set. These frameworks are expected to provide clearer models for concepts involving greater uncertainty and ambiguity.

Looking ahead, we anticipate further development of extended frameworks based on HyperFuzzy Sets [380–383], Hyper-Neutrosophic Sets [384–386], Bipolar Neutrosophic Sets [15, 387, 388], HyperSoft Sets [389–391], Rough Sets [392–394], Plithogenic Sets [42, 44, 395], and Functorial Sets [396].

In addition, we believe that further extensions employing HyperFunctions and SuperHyperFunctions [397–401] can also be explored. Since this study focuses on theoretical aspects, future work may include computational and experimental research utilizing machine learning and deep learning methods.

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Conflicts of Interest

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Data Availability

This paper is theoretical and did not generate or analyze any empirical data. We welcome future studies that apply and test these concepts in practical settings.

Research Integrity

The author confirms that this manuscript is original, has not been published elsewhere, and is not under consideration by any other journal.

Use of Computational Tools

All proofs and derivations were performed manually; no computational software (e.g., Mathematica, SageMath, Coq) was used.

Code Availability

No code or software was developed for this study.

Ethical Approval

This research did not involve human participants or animals, and therefore did not require ethical approval.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Supplementary Information

No supplementary materials accompany this paper.

Disclaimer

The ideas presented here are theoretical and have not yet been validated through empirical testing. While we have strived for accuracy and proper citation, inadvertent errors may remain. Readers should verify any referenced material independently. The opinions expressed are those of the authors and do not necessarily reflect the views of their institutions.

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